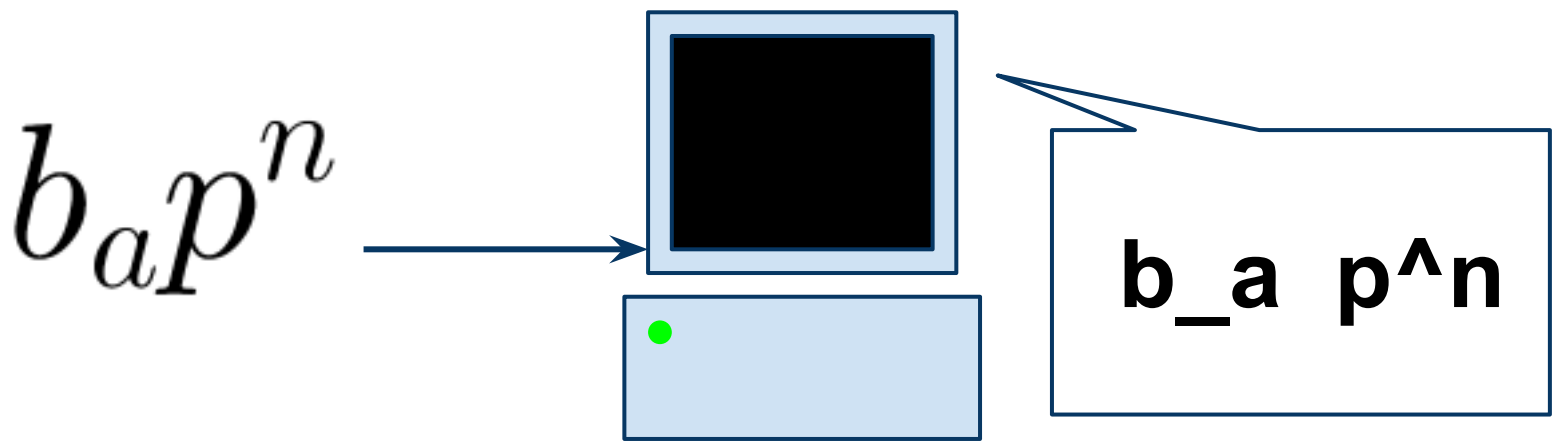
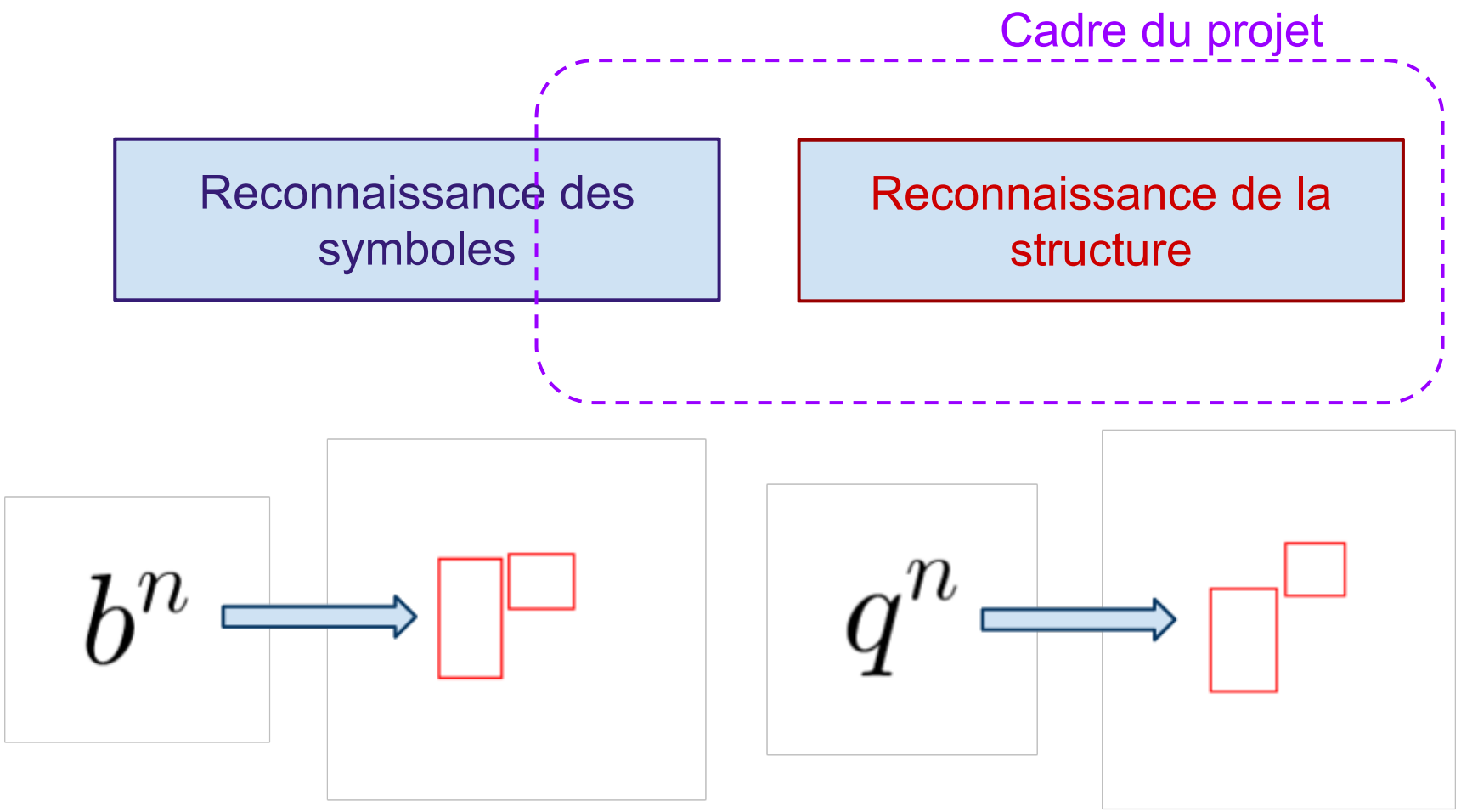
Une approche pour la reconnaissance *a priori* de la structure d'expressions mathématiques

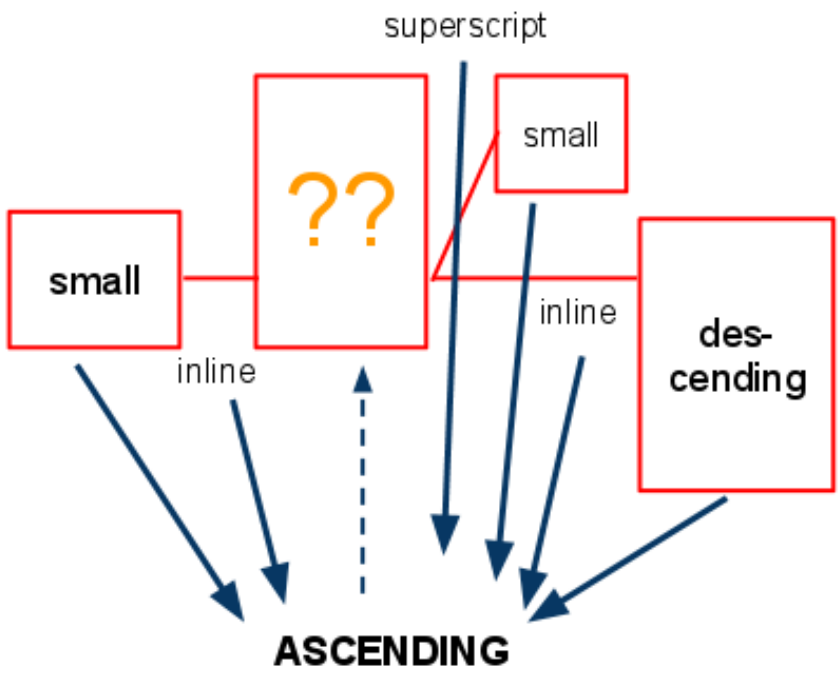


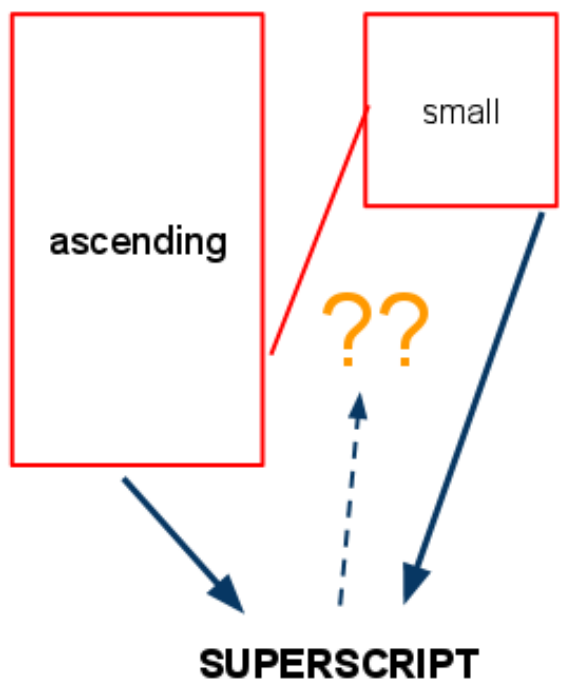
Théodore Bluche
Superviseur: Vasile Palade

Cadre du projet et motivations



Hypothèse

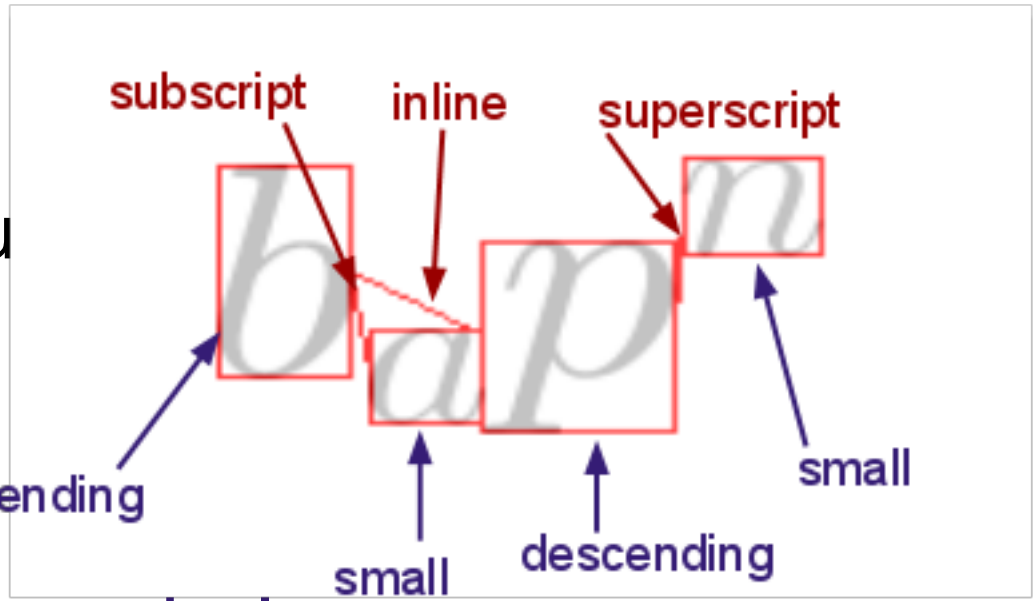

superscript
small
??
small
inline
inline
des-
cending
ASCENDING


small
ascending
??
SUPERSCRIPT

Deux tâches

Déterminer les relations entre les symboles

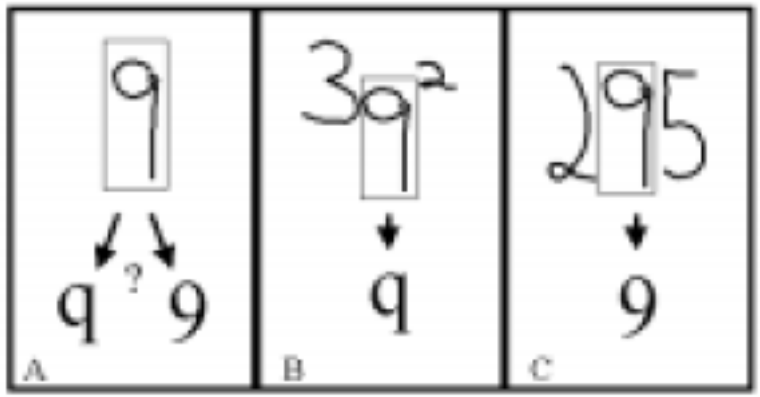
- indice, exposant ...
- on peut s'aider des classes des symboles en jeu



Déterminer les classes des symboles

- ascendant (t, b, k, ...), descendant (p, y, ...), petit (a, s, n, ...), ...
- on peut utiliser le contexte (*e.g.* la position relative de l'exposant)

Motivations de l'approche proposée

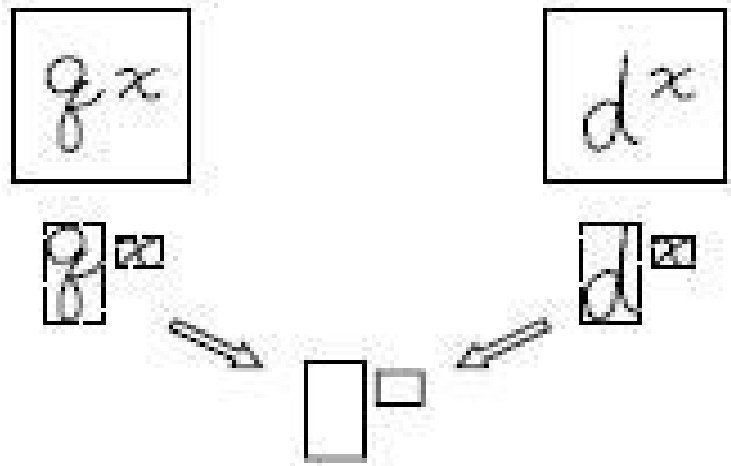


Le problème "q-9"

Erik G. Miller and Paul A. Viola.
Ambiguity and constraint in mathematical expression recognition. In AAAI '98/IAAI '98: Proceedings of the fifteenth national/tenth conference on Artificial intelligence/Innovative applications of artifcial intelligence. 1998.

Disposition similaire dans des contextes différents

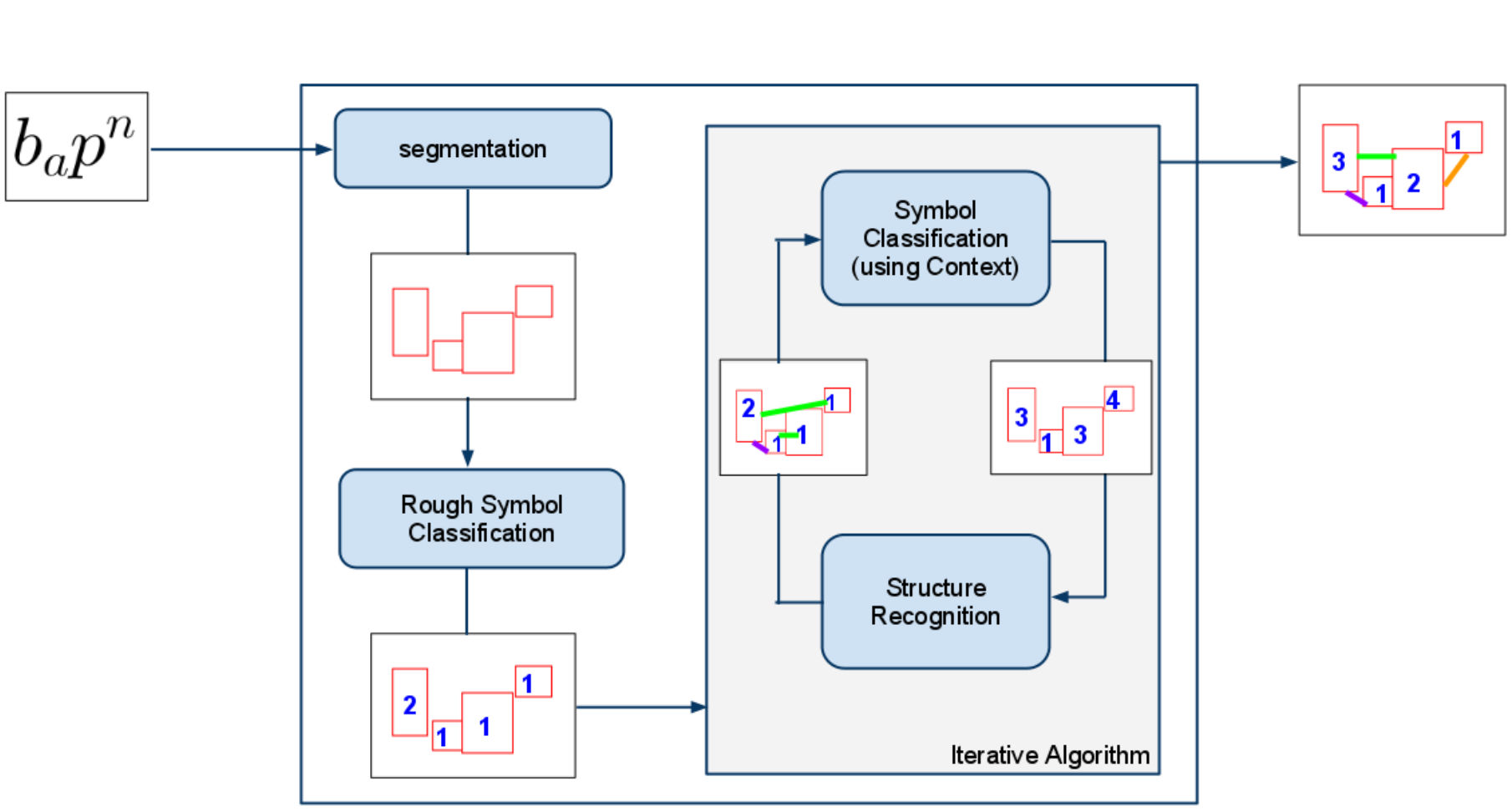
Kam-Fai Chan and Dit-Yan Yeung.
Mathematical expression recognition: A survey.
In International Journal on Document Analysis and Recognition, 1999.



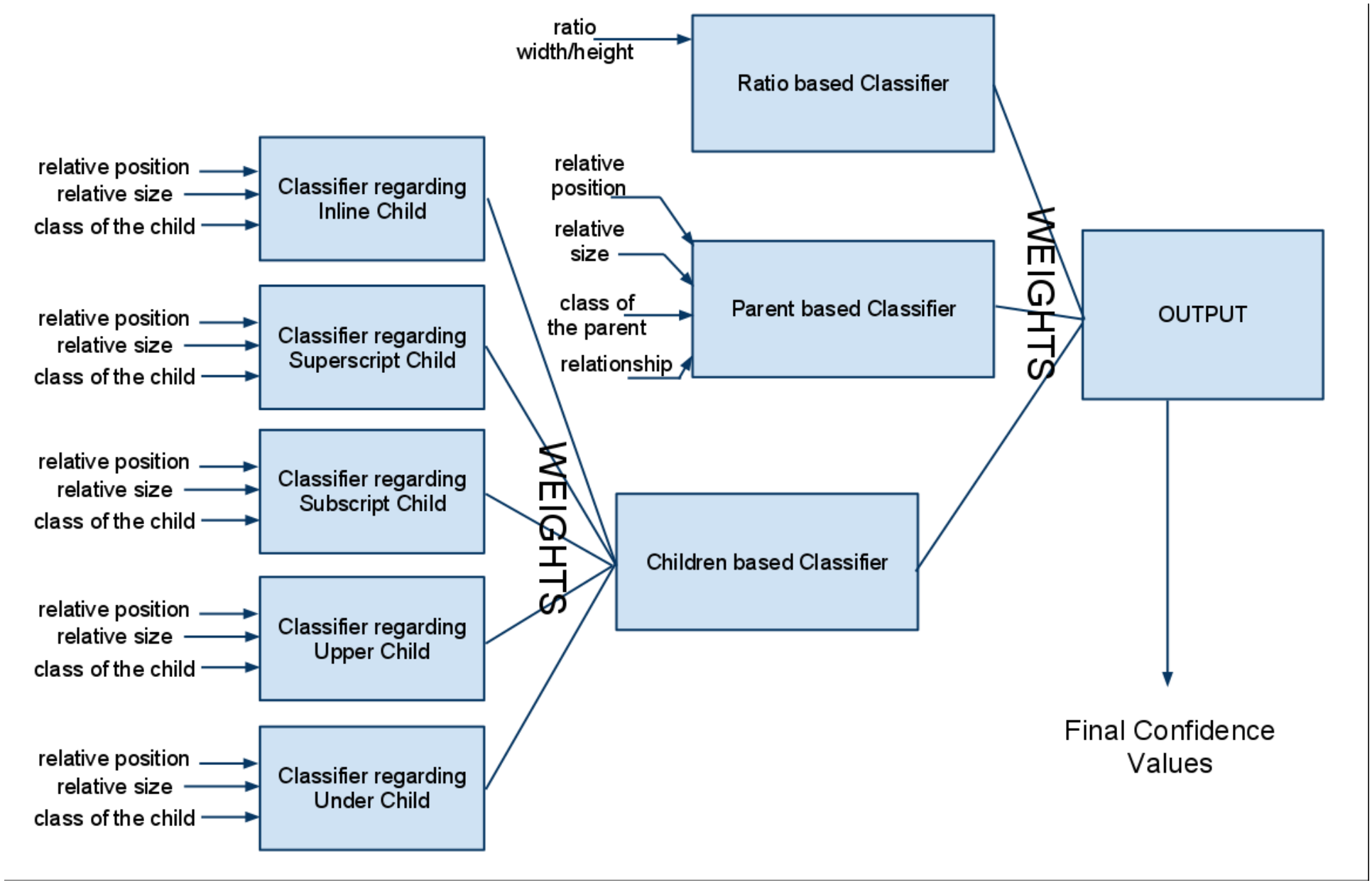
Hypothèses simplificatrices

- On n'utilise qu'un nombre restreint de symboles et de relations
- On se concentre principalement sur des équations produites par un ordinateur (type Latex)
- On suppose que tous les symboles sont des composantes connexes
- On suppose qu'un symbole fils (indice, exposant, ...) ne se trouvera jamais plus à gauche que son père

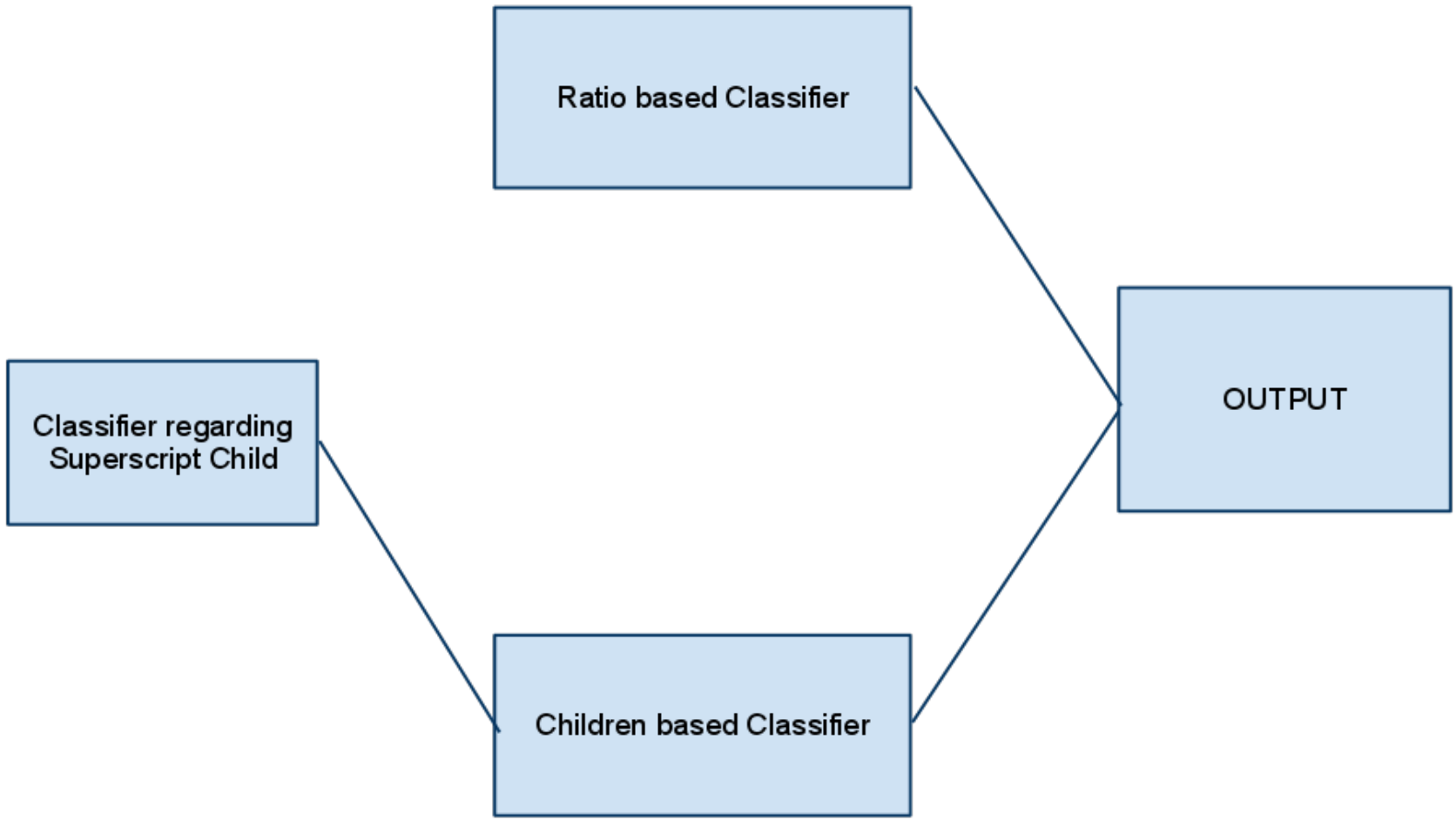
Méthode choisie



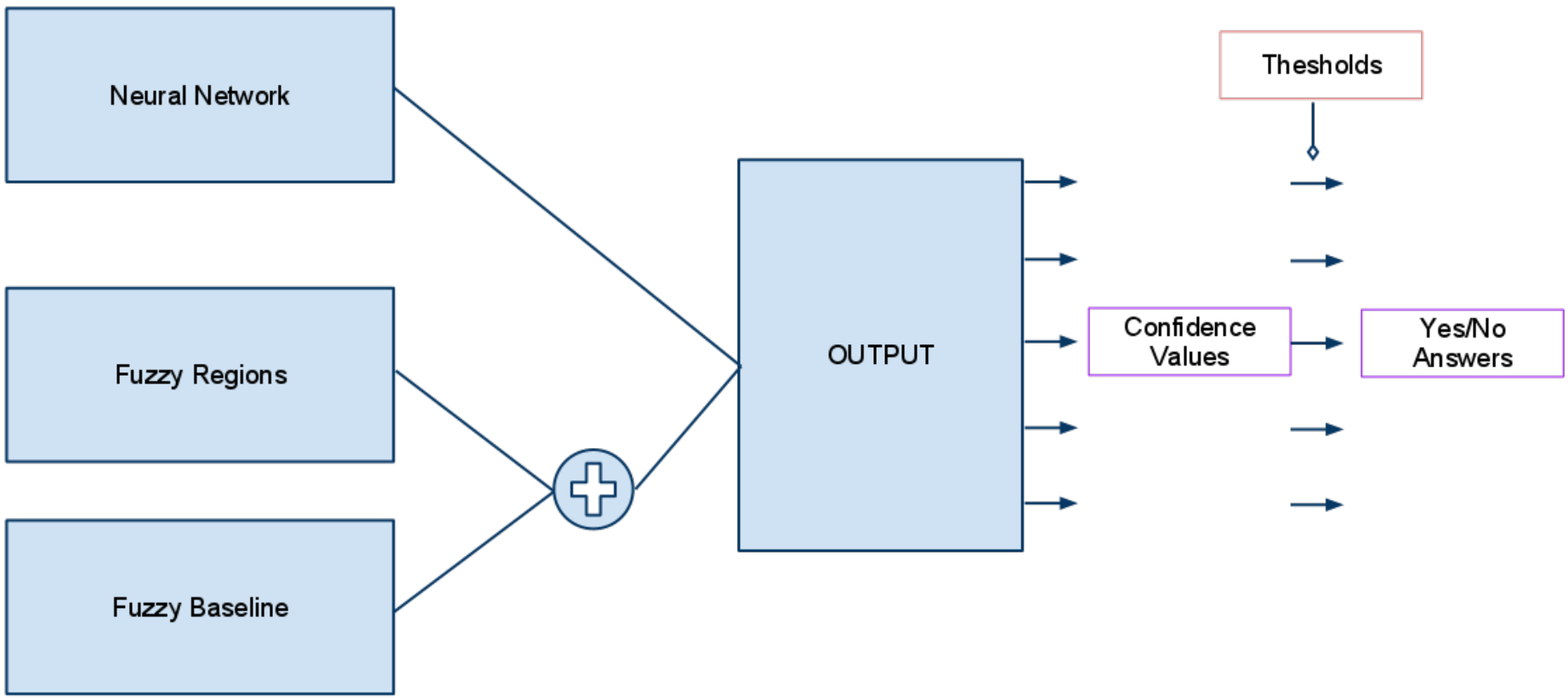
Classification des symboles



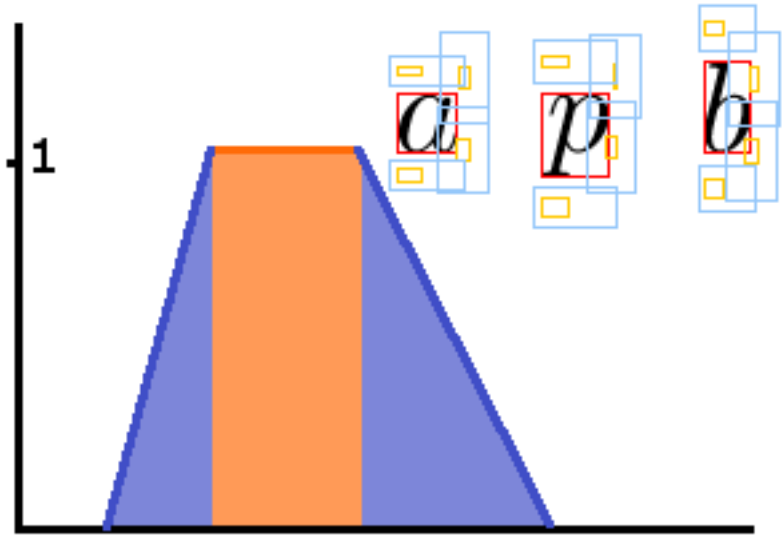
Manque de contexte



Classification des relations



Fuzziness

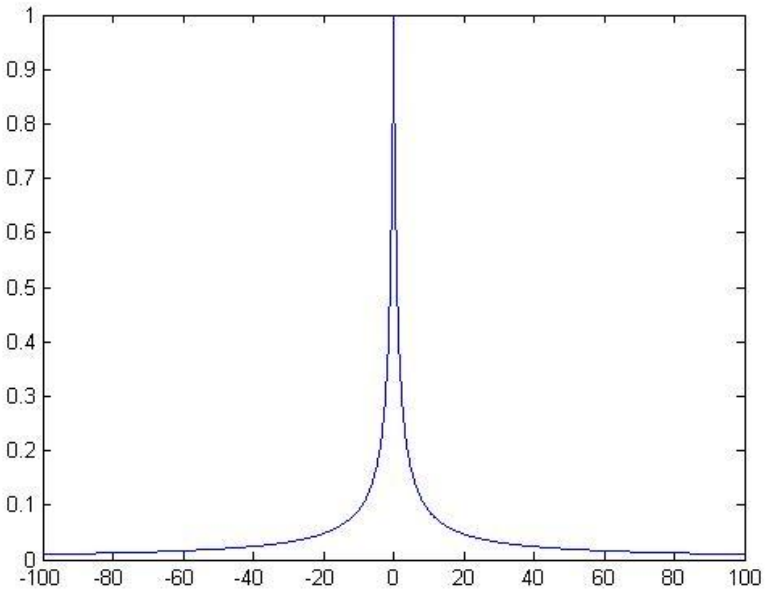


Régions

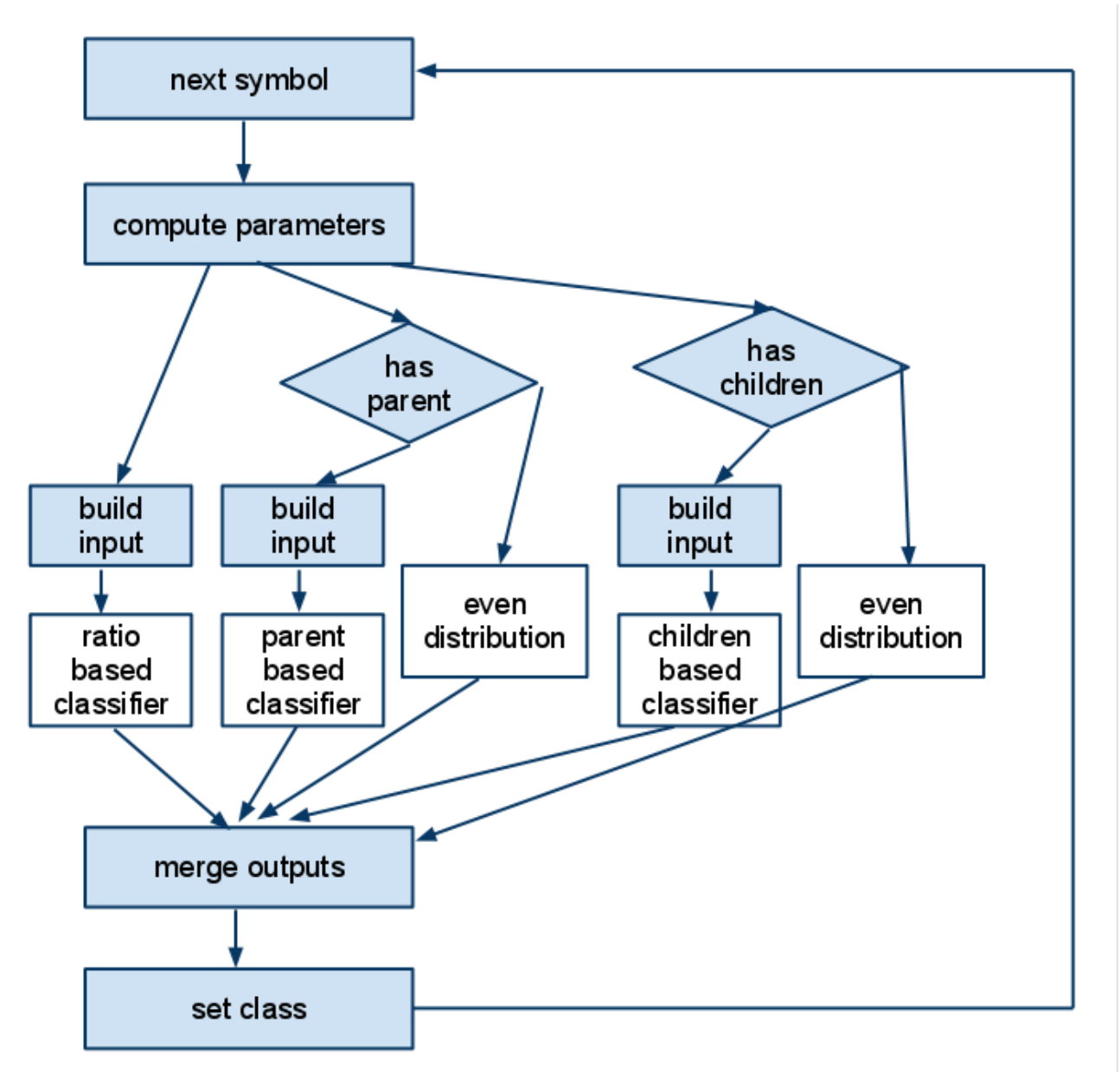
Régions construites "à la main" à partir des statistiques issues du jeu de données

Lignes de base

Score inversement proportionnel à la distance à la ligne de base "mère"



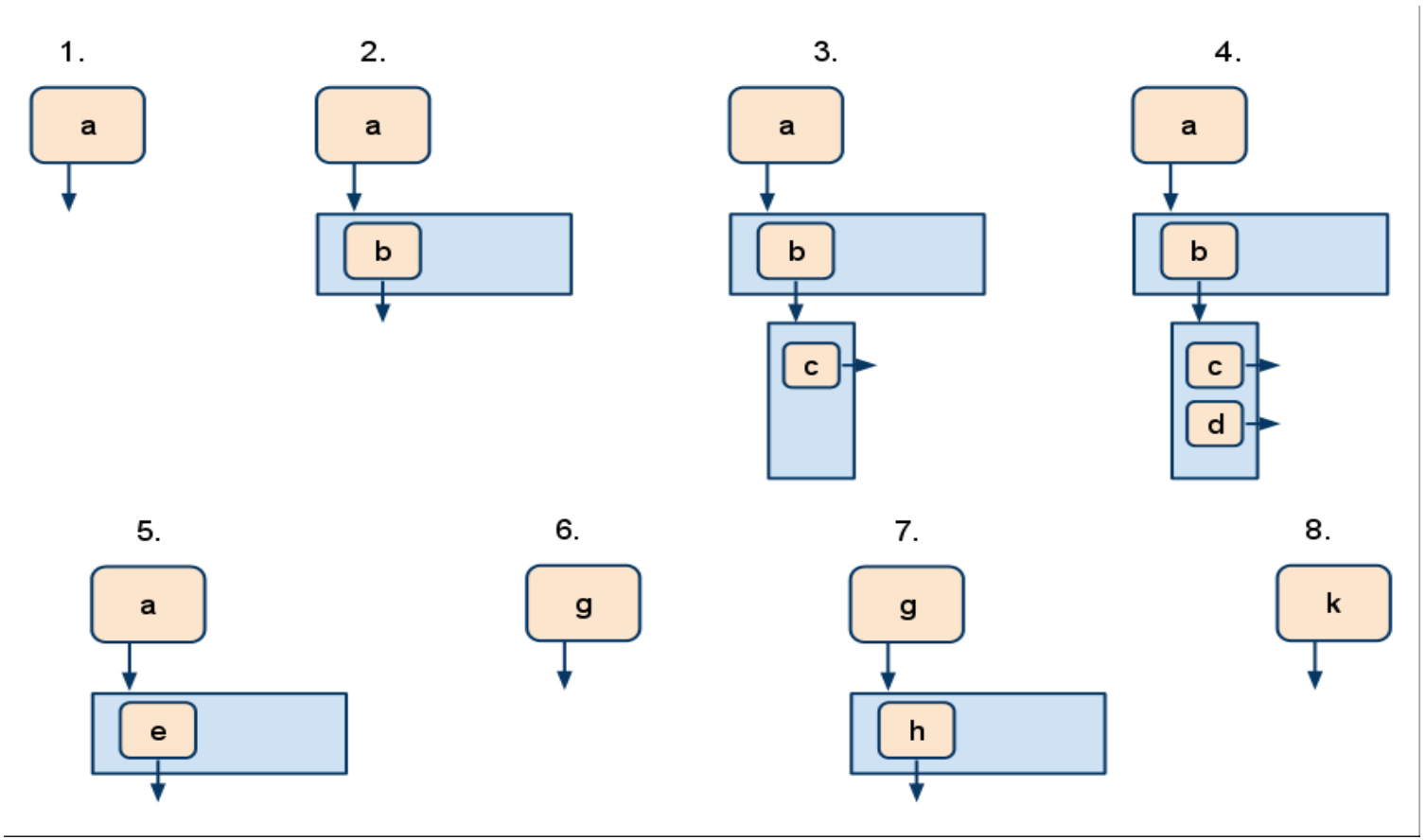
Processus de reconnaissance



Classification des symboles

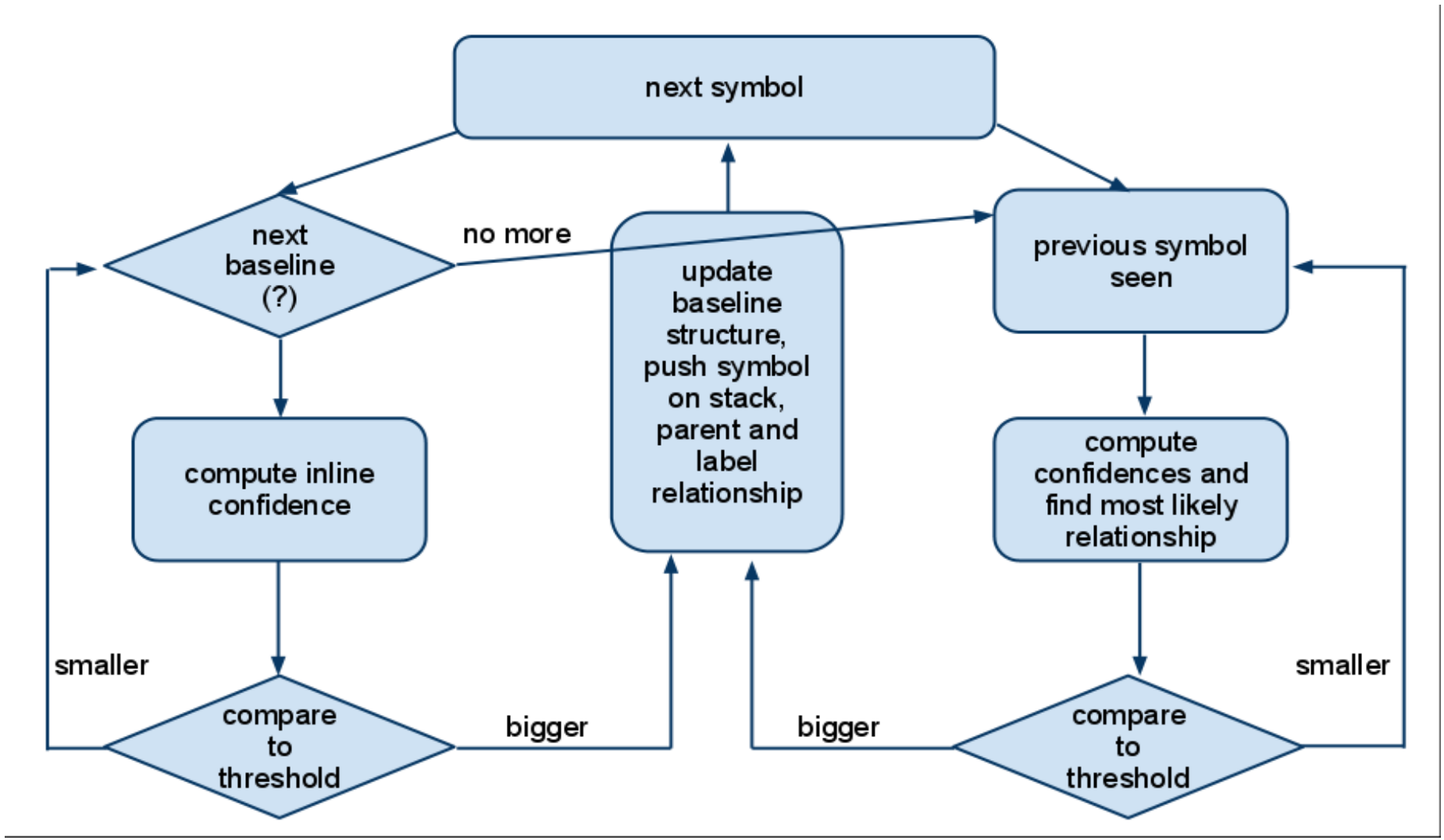
Processus de reconnaissance

$$a^{b^{c}_{d}e}g_{h}k$$



Classification des relations (1/2)

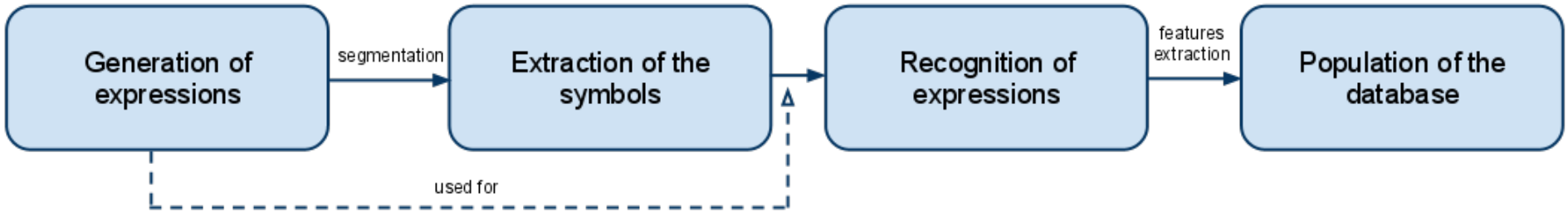
Processus de reconnaissance



Classification des relations (2/2)

Jeux de données

Pour la phase d'apprentissage

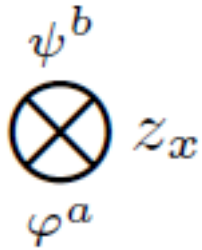


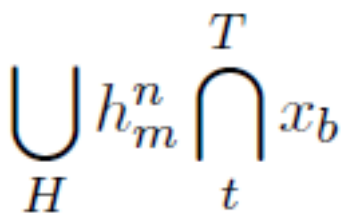
- Génération automatique d'un jeu de données étiquetable automatiquement
- Ajout d'une variation virtuelle sur la position et la taille des symboles
- 7316 exemples créés

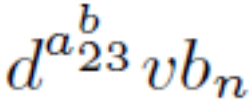
Jeux de données

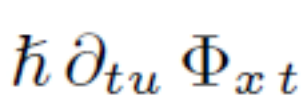
Pour la phase de tests

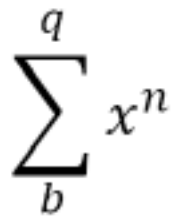
- Création de six jeux de données contenant 94 expressions mathématiques fictives
- Ils contiennent des expressions de complexité différentes, avec des styles différents
- Etiquetage à la main

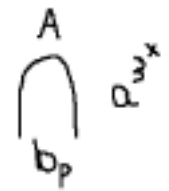












Evaluation des performances

Indicateurs

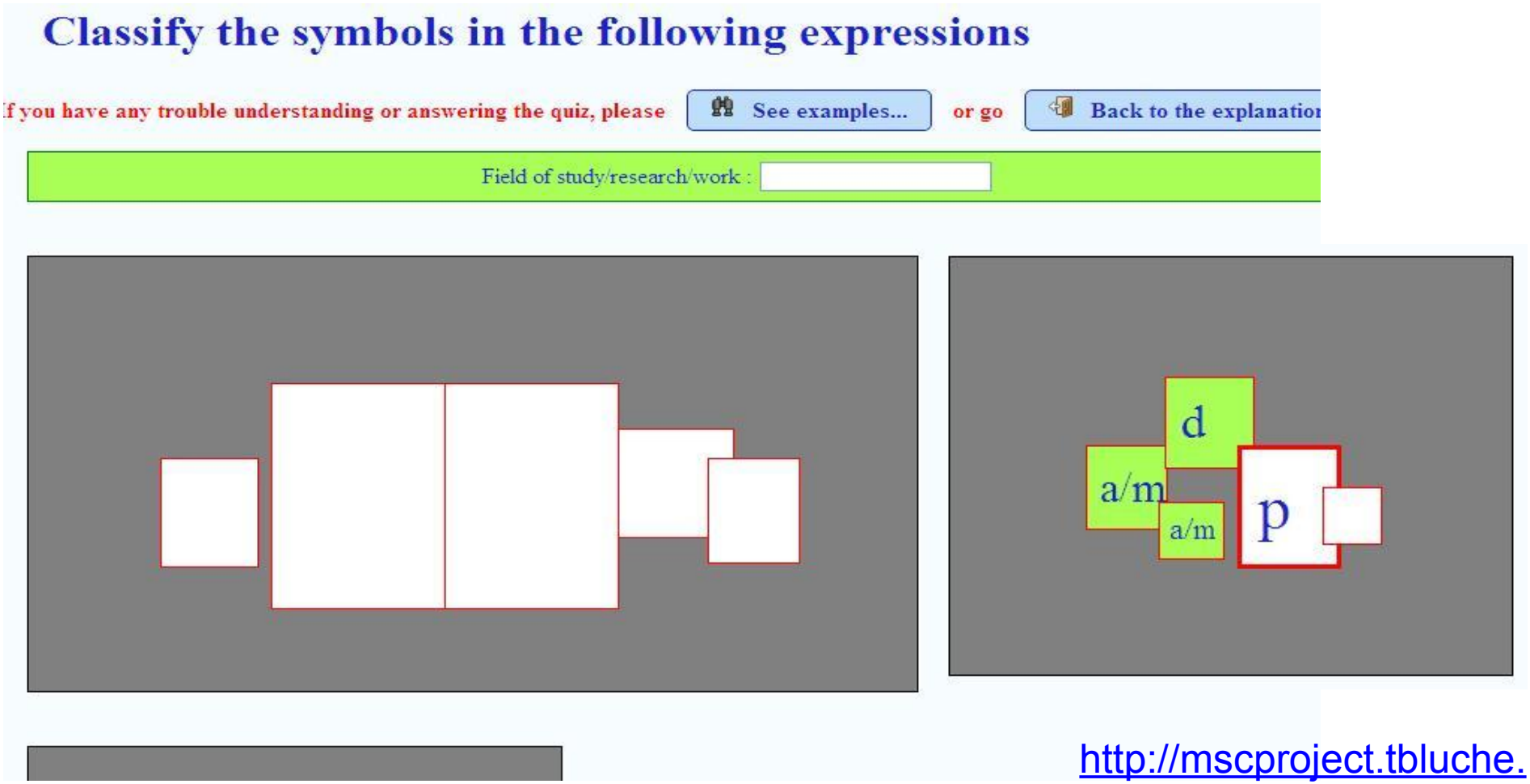
- ***Erreurs***
  - Erreur de parent (*Ep*) : proportion de symboles associés au mauvais père
  - Erreur de symbole (*Es*) : proportion de symboles associés à la mauvaise classe
  - Erreur de relation (*Er*) : proportion de mauvais étiquetage de relations

- ***Justesse*** *(correctness)*
  - Justesse de symbole (*Cs*) : score du symbole dans la "vraie" classe
  - Justesse de relation (*Cr*) : score de la relation dans la "vraie" classe

Evaluation des performances

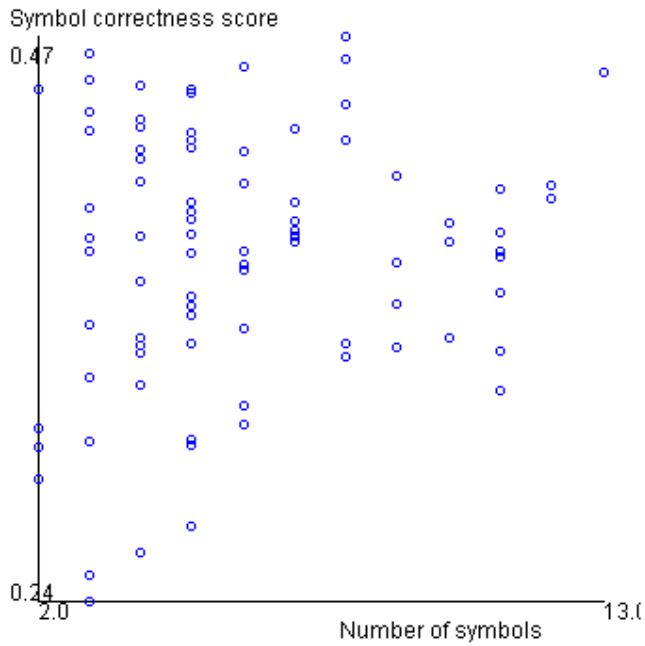
Comparaison à l'étiquetage par un humain

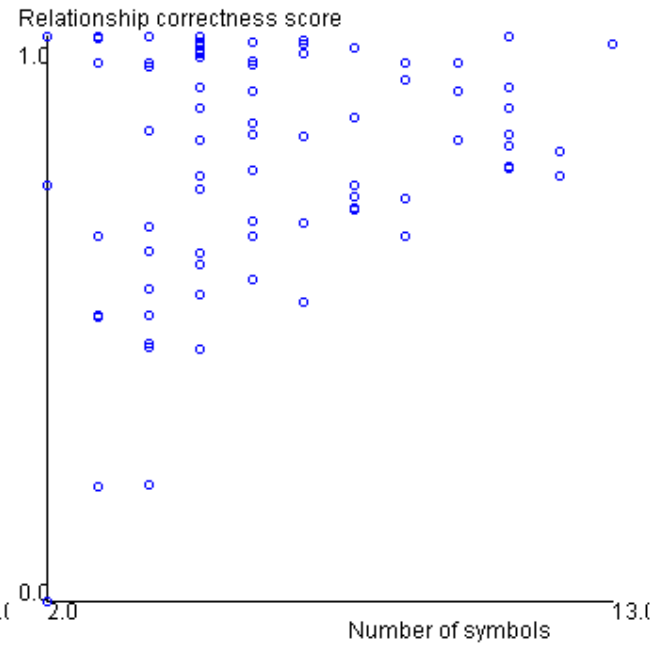
Sous forme d'un quizz internet (101 réponse obtenues)

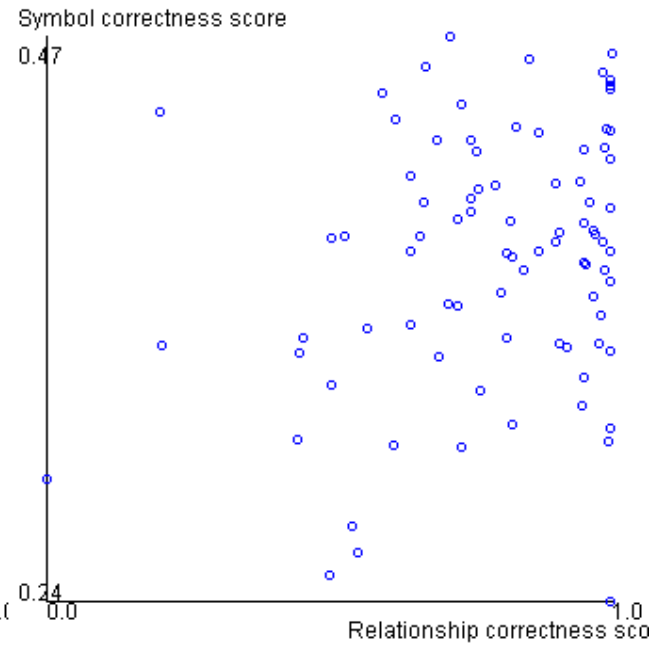


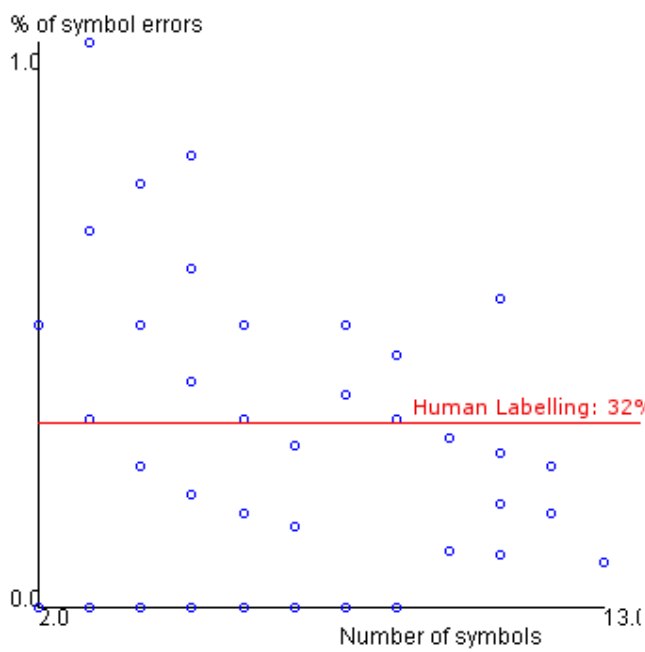
http://mscproject.tbluche.com

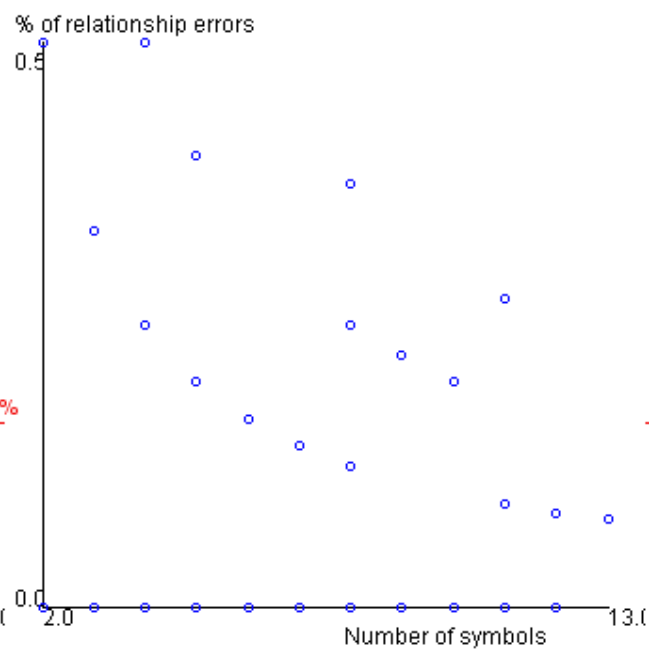
Résultats

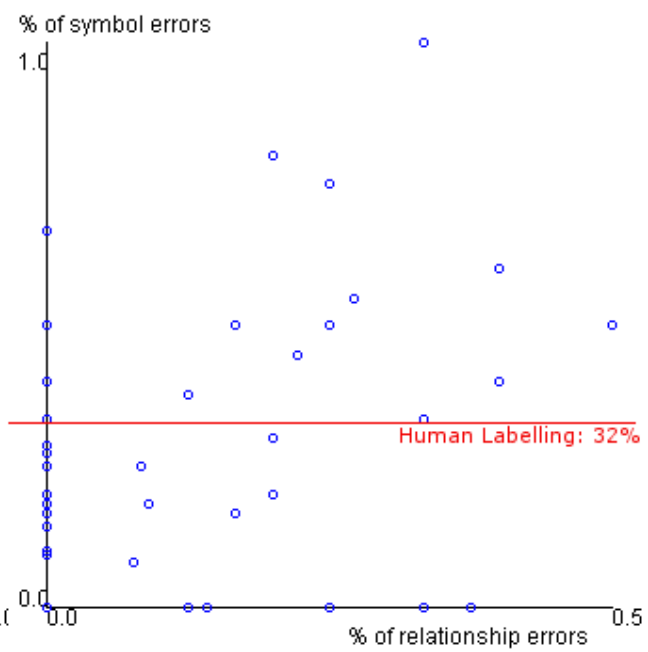

Symbol correctness score
0.47
0.24
2.0
Number of symbols


Relationship correctness score
1.0
0.0
2.0
Number of symbols


Symbol correctness score
0.47
0.24
0.0
1.0
Relationship correctness sco


% of symbol errors
1.0
0.0
2.0
Human Labelling: 32%
Number of symbols


% of relationship errors
0.5
0.0
2.0
Number of symbols


% of symbol errors
1.0
0.0
0.0
0.5
Human Labelling: 32%
% of relationship errors

Résultats

Table 6.3: Test Sets Scores

Testset	Symbol Classi-fication (%)	Parenting (%)	Relationship Recognition (%)	Symbol Correctness	Relationship Correctness
testset0	66.67	95.24	84.52	0.362	0.537
testset1	87.78	95.04	95.04	0.404	0.922
testset2	81.63	95.92	90.48	0.392	0.777
testset3	78.79	90.91	91.67	0.390	0.922
testsetNL	61.02	94.92	89.84	0.347	0.826
testsetHW	70.37	96.30	92.59	0.381	0.852

Questions ?

Paramètres

Classifieur de symboles

Table 4.5: Parameters of the Symbol Classifiers

Classifier	Parameters	Number of examples	Accuracy
Parent-based	Learning rate: 0.3 Momentum: 0.2 Epochs: 500	2981	72.2%
Inline child based	Learning rate: 0.3 Momentum: 0.2 Epochs: 500	1553	70%
Other child based	Learning rate: 0.3 Momentum: 0.2 Epochs: 500	About 800	About 98%

Ratio : 0.35
Parent : 0.10
Enfants : 0.55

(Chaque enfant : 0.20)

Paramètres

Classifieur de relations - NN

- H - the relative size of the child to its parent: $H = \frac{h_p}{h_c}$
- D - the relative vertical position of the child: $D = \frac{y_p - y_c}{h_p}$
- V - the relative horizontal position of the child: $V = \frac{xmax_p - xmin_c}{w_p}$
- Classe du père potentiel
- Classe de l'enfant potentiel

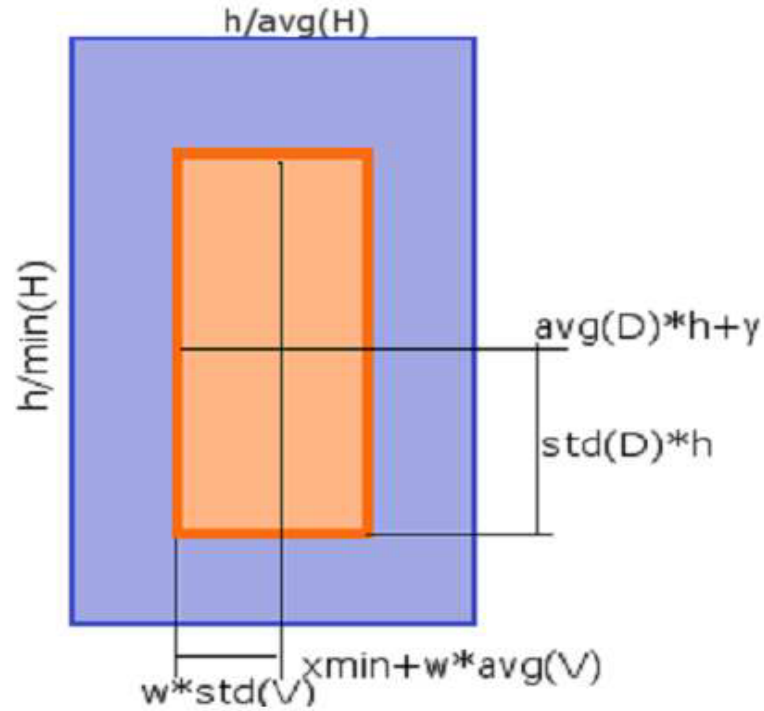
Learning rate : 0.2
Momentum : 0.2
Epochs : 500

Paramètres

Fuzzy regions

$$
\begin{aligned}
x_{min} &= x - w * (\overline{V} + \sigma_V) \\
x_{max} &= x - w * (\overline{V} - \sigma_V) \\
y_{min} &= m - h * (\overline{D} + \sigma_D) \\
y_{max} &= m - h * (\overline{D} - \sigma_D)
\end{aligned}
$$

$$
\begin{aligned}
h_{marg} &= \frac{1}{2}(\frac{h}{H_{min}} - h_{crisp}) \\
v_{marg} &= \frac{1}{2}(\frac{h}{\overline{H}} - w_{crisp})
\end{aligned}
$$



Fuzzy baselines

$$b_1 = \frac{\alpha}{\alpha + d}$$

Possible child classifier

Arbre de décision J48

Paramètres

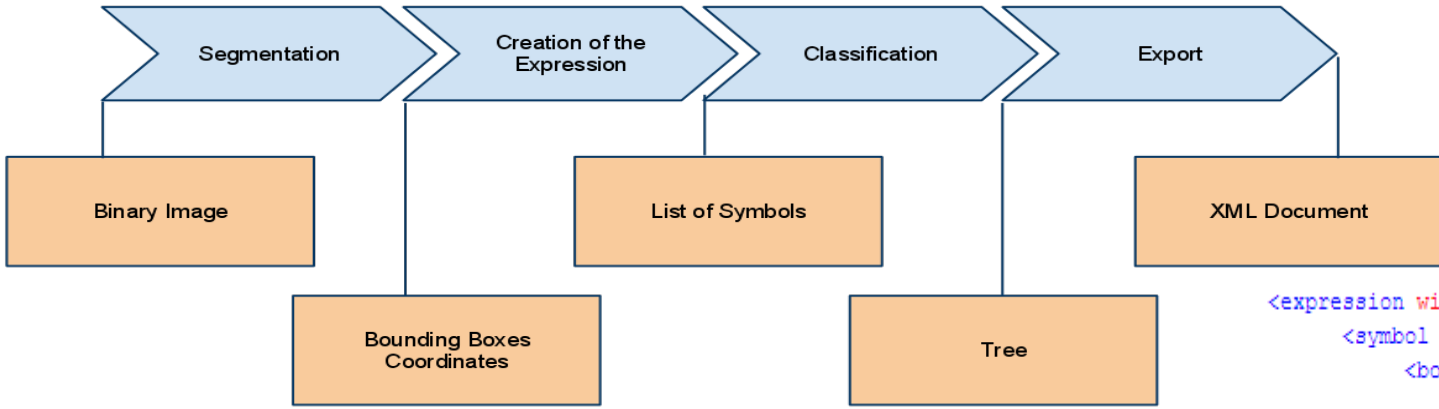
En ligne

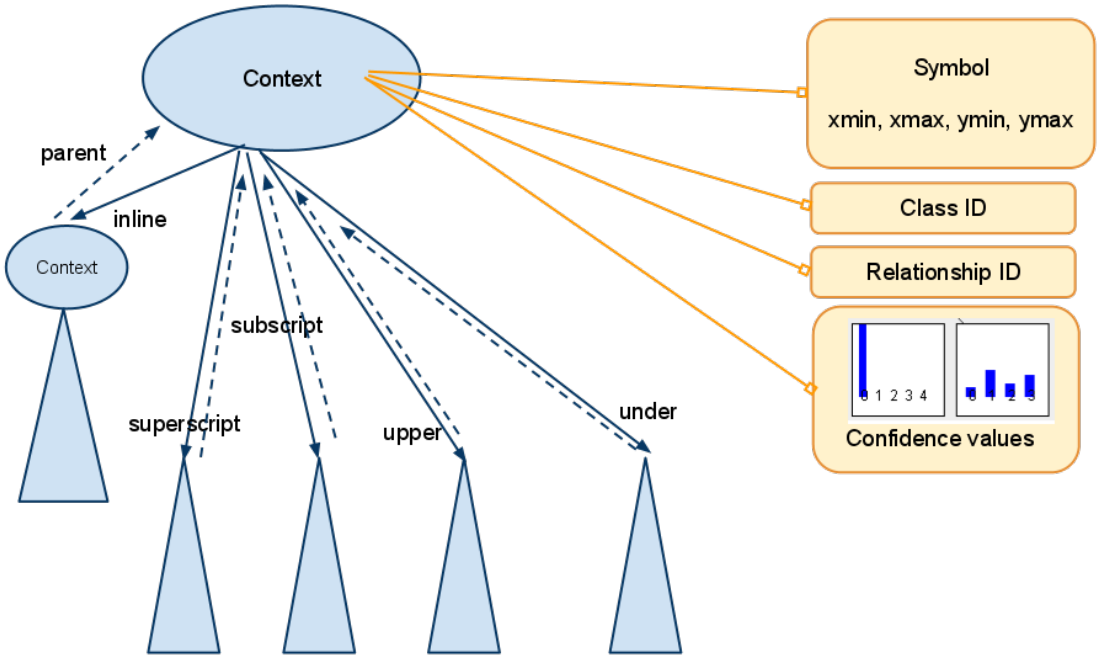
- NN : 1
- PCC : 1
- FR : 0
- FB : 4

Autres

- NN : 1
- PCC : 2
- FR : 4
- FB : 0

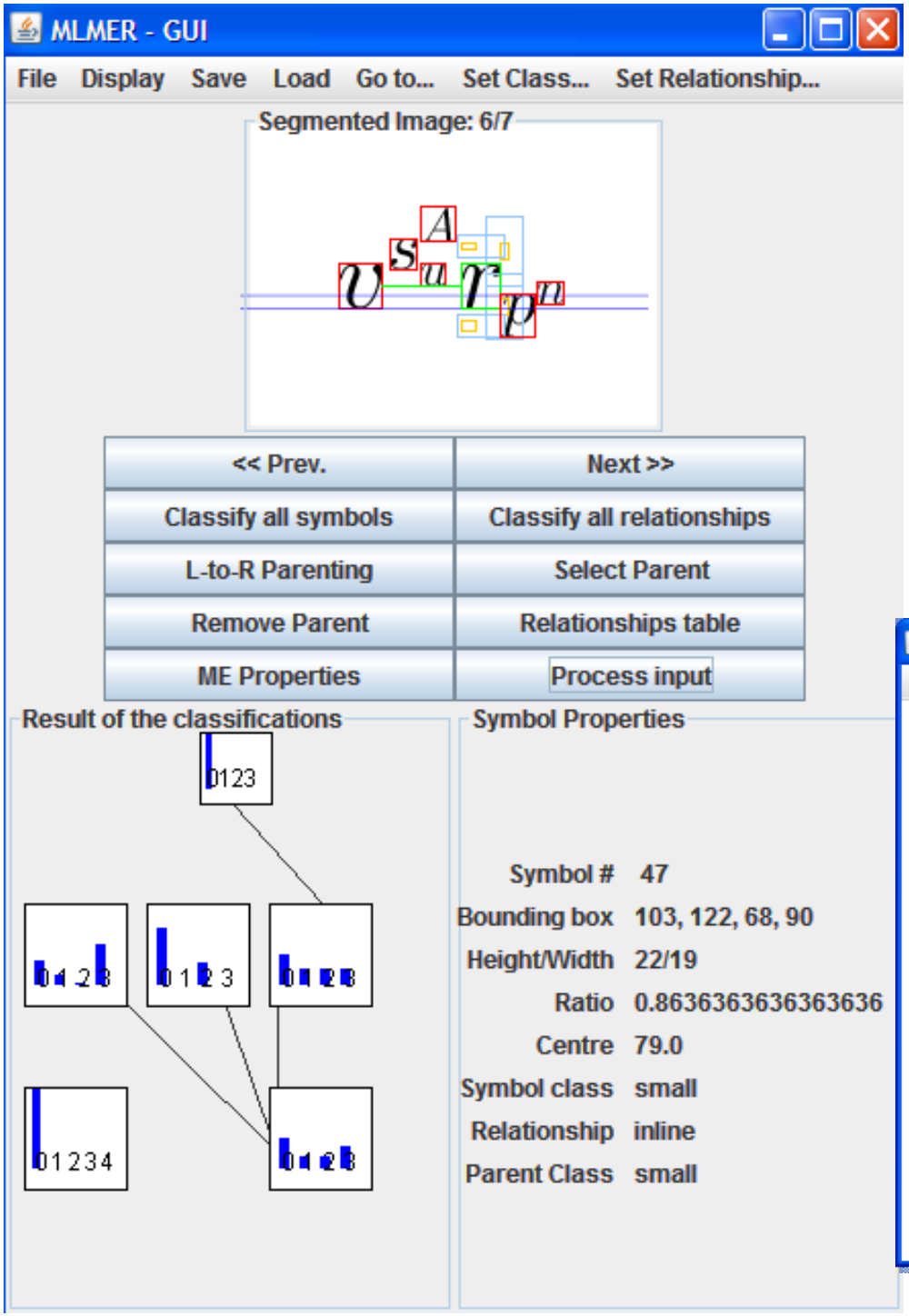
Implementation - divers

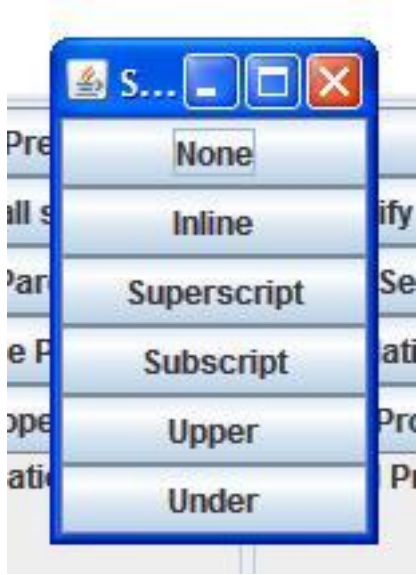


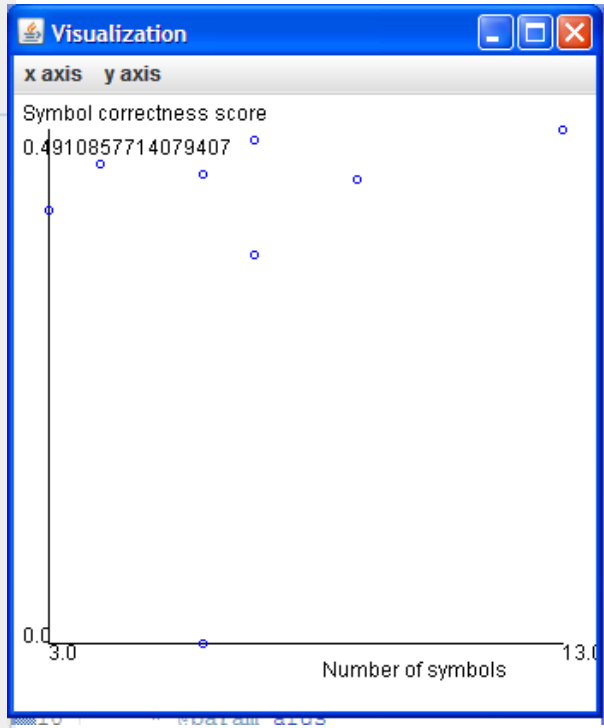


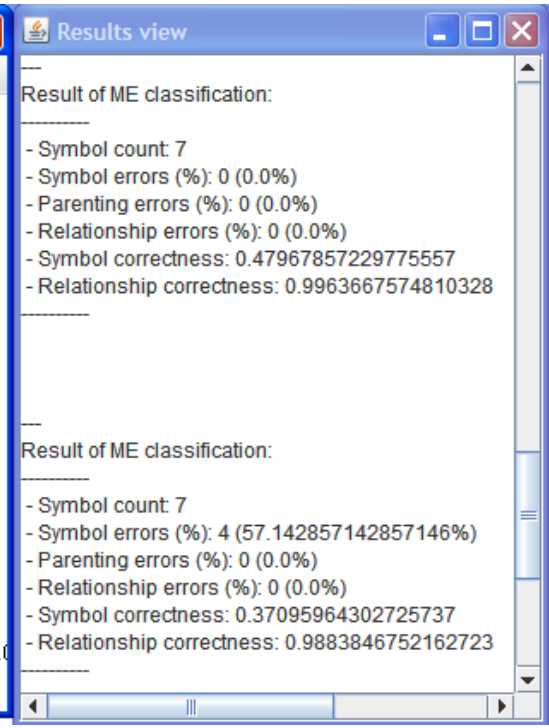

```
<expression width="126" height="129">
    <symbol id="6" class="1" rel="-1">
        <boundingBox>
            <xmin>43</xmin>
            <xmax>65</xmax>
            <ymin>63</ymin>
            <ymax>85</ymax>
        </boundingBox>
        <symbolClass>
            <class id="1">0.4303918844197614</class>
            <class id="2">0.1428140109883948</class>
            <class id="3">0.26488460479316966</class>
            <class id="4">0.16190949979867414</class>
        </symbolClass>
        <relationshipClass>
            <class id="0">0.0</class>
            <class id="1">0.0</class>
            <class id="2">0.0</class>
            <class id="3">0.0</class>
            <class id="4">0.0</class>
        </relationshipClass>

        <!--CHILDREN-->
        <symbol id="5" class="3" rel="1">
        ...
        </symbol>
        <symbol id="3" class="2" rel="0">
        ...
        </symbol>
        ...
```

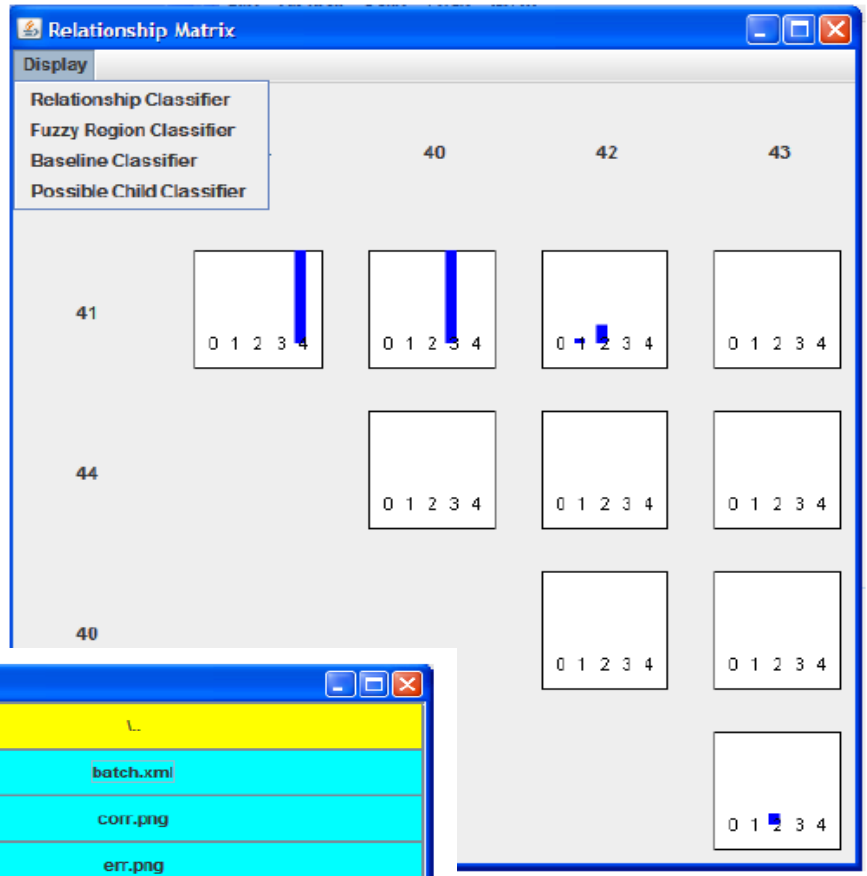
Interface


MLMER - GUI
File
Display
Save
Load
Go to...
Set Class...
Set Relationship...
Segmented Image: 6/7
<< Prev.
Next >>
Classify all symbols
Classify all relationships
L-to-R Parenting
Select Parent
Remove Parent
Relationships table
ME Properties
Process input
Result of the classifications
0123
01234
Symbol Properties
Symbol # 47
Bounding box 103, 122, 68, 90
Height/Width 22/19
Ratio 0.8636363636363636
Centre 79.0
Symbol class small
Relationship inline
Parent Class small


None
Inline
Superscript
Subscript
Upper
Under


Visualization
x axis
y axis
Symbol correctness score
0.4910857714079407
0.0
3.0
Number of symbols


Results view
Result of ME classification:
- Symbol count: 7
- Symbol errors (%): 0 (0.0%)
- Parenting errors (%): 0 (0.0%)
- Relationship errors (%): 0 (0.0%)
- Symbol correctness: 0.47967857229775557
- Relationship correctness: 0.9963667574810328
Result of ME classification:
- Symbol count: 7
- Symbol errors (%): 4 (57.142857142857146%)
- Parenting errors (%): 0 (0.0%)
- Relationship errors (%): 0 (0.0%)
- Symbol correctness: 0.37095964302725737
- Relationship correctness: 0.9883846752162723



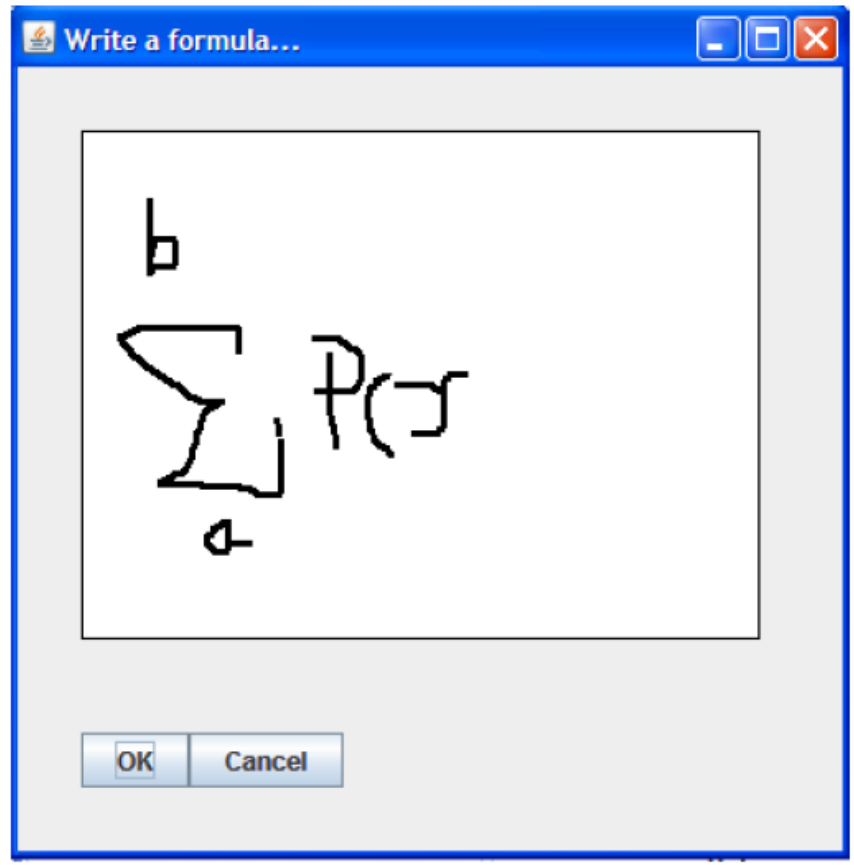


Figure 5.11: Drawing Window

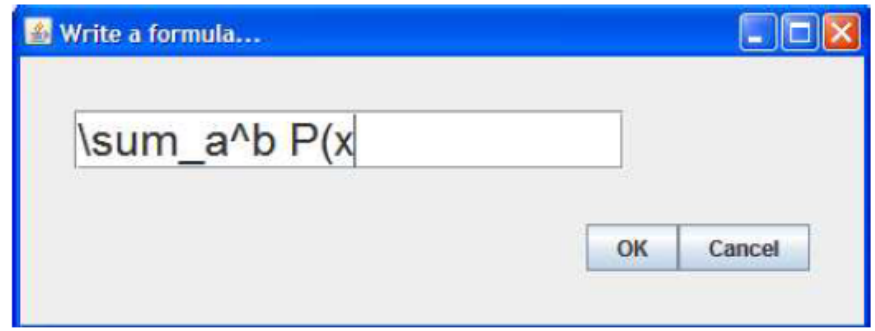


Figure 5.12: Latex Command Window

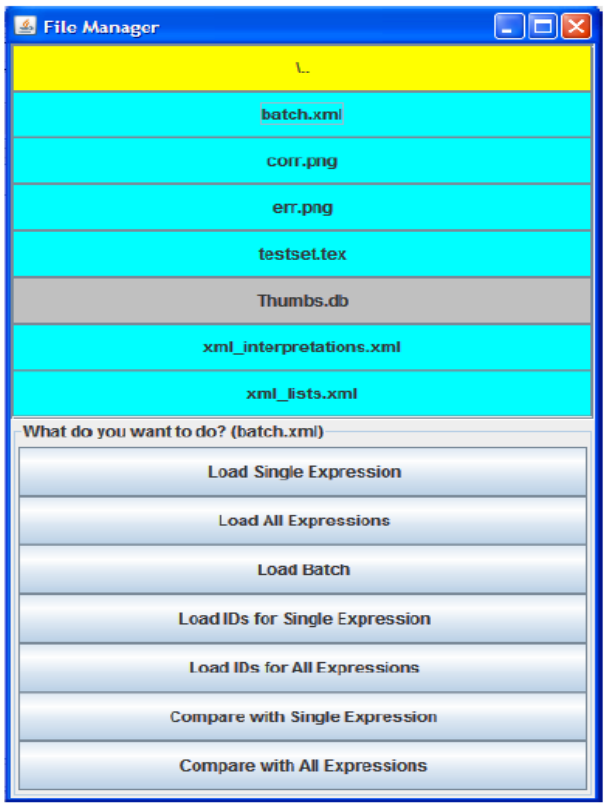
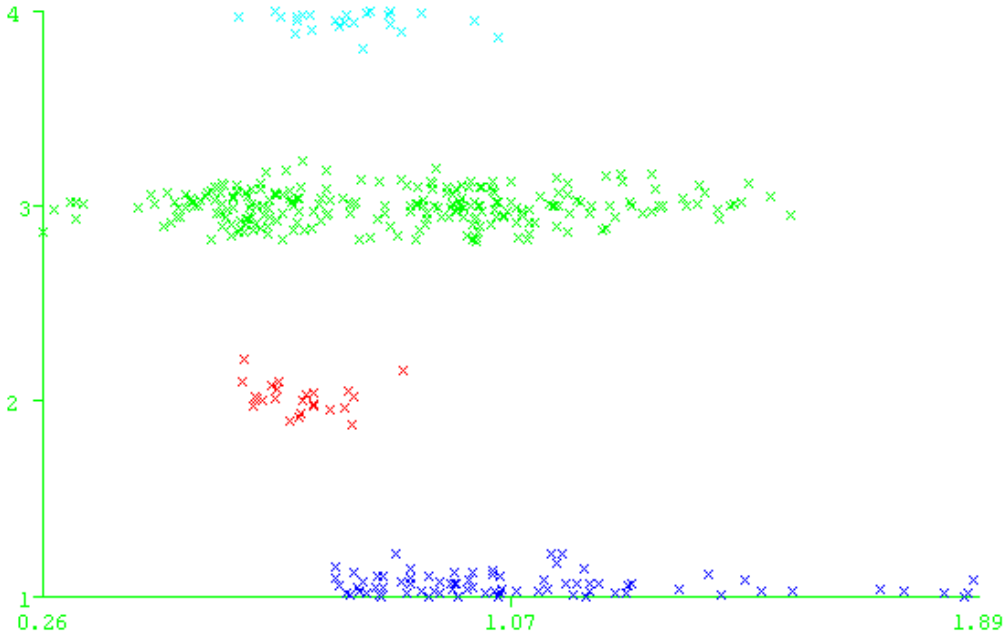


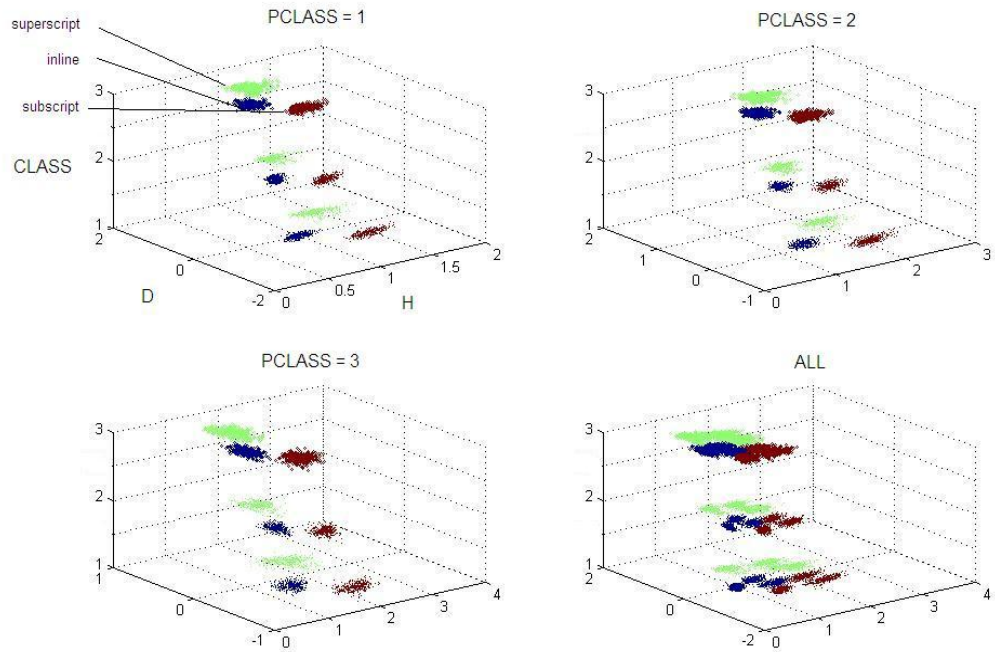
Figure 5.13: File Manager

Analyse des données



Ratio largeur/hauteur

Relations, selon la classe du symbole père et du symbole enfant



Cadre du projet

Table 3.1: Symbol classes used

Symbol Class	Examples
1 - Small symbols	a, e, r, u, o, s, m, x, c, n, ...
2 - Descending symbols	y, p, q, g, ...
3 - Ascending symbols	A-Z, 0-9, t, d, h, k, l, ...
4 - Variable range symbols	$\sum, \prod, \bigcup, \bigcap, \ldots$

Table 3.2: Relationships used

Relationship Class	Examples
0 - Inline	xy, tan, 42, $10x$, $\sum n$, ...
1 - Superscript	p^n, b^a, x^y, ...
2 - Subscript	p_a, b_n, x_3, ...
3 - Upper	$\sum^{N}, \ldots$
4 - Under	$\prod_{x}, \ldots$

Résultats détaillés

Actual Expression	Expected Result	Recognition Result	Statistics
$nocontext$	$aaaaabaab$	$aaaaabaab$	$N = 9$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.41$, $C_r = 0.64$
dc	ba	pa	$N = 2$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.3$, $C_r = 0.73$
vlm	aba	aba	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.44$, $C_r = 0.2$
EPx	bba	bba	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.39$, $C_r = 0.5$
123	bbb	bbb	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.47$, $C_r = 1.0$
$1a2b3p$	$babbbp$	$babbbp$	$N = 6$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.46$, $C_r = 0.67$
$tan\pi$	$baaa$	$baaa$	$N = 4$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.44$, $C_r = 0.61$
$\sum a$	$\sum a$	b_a	$N = 2$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 50\%$ $C_s = 0.29$, $C_r = 0.0$
$\sum pacb$	$\sum paab$	$a_{b^a \sum b}$	$N = 5$ $e_s = 60\%$, $e_p = 20\%$, $e_r = 40\%$ $C_s = 0.3$, $C_r = 0.44$
$\prod a \bigcup N$	$\sum a \sum b$	$\sum ap_a$	$N = 4$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 25\%$ $C_s = 0.34$, $C_r = 0.45$

Actual Expression	Expected Result	Recognition Result	Statistics
$\bigcap \bigcup Y$	$\sum \sum b$	pp_a	$N = 3$ $e_s = 100\%$, $e_p = 0\%$, $e_r = 33\%$ $C_s = 0.25$, $C_r = 0.5$
$\prod 25nB$	$\sum bbab$	$\sum bbaa$	$N = 5$ $e_s = 20\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.75$
$5 \sum gh$	$b \sum pb$	ba_{pp}	$N = 4$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 50\%$ $C_s = 0.26$, $C_r = 0.55$
$a \sum n \prod wp$	$a \sum a \sum ap$	$ab_a \sum ap$	$N = 6$ $e_s = 16\%$, $e_p = 16\%$, $e_r = 16\%$ $C_s = 0.35$, $C_r = 0.57$
$PadV$	$babb$	$ba^P a$	$N = 4$ $e_s = 50\%$, $e_p = 25\%$, $e_r = 25\%$ $C_s = 0.35$, $C_r = 0.45$
$PV\ nRT$	$bbabb$	$bbaaa$	$N = 5$ $e_s = 40\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.36$, $C_r = 0.72$
$\alpha\Gamma\pi\Delta$	$abab$	$b^{\sum} aa$	$N = 4$ $e_s = 75\%$, $e_p = 25\%$, $e_r = 25\%$ $C_s = 0.33$, $C_r = 0.5$
$\bigwedge \alpha p$	$\sum ap$	p_{ap}	$N = 3$ $e_s = 33\%$, $e_p = 0\%$, $e_r = 33\%$ $C_s = 0.35$, $C_r = 0.64$
$\sigma \bigcup Au$	$a \sum ba$	ap_{aa}	$N = 4$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 25\%$ $C_s = 0.34$, $C_r = 0.2$
$*\varphi \bigvee \bigwedge T\ p$	$p \sum \sum bp$	$\sum pp_{ap}$	$N = 5$ $e_s = 80\%$, $e_p = 0\%$, $e_r = 20\%$ $C_s = 0.27$, $C_r = 0.54$

Actual Expression	Expected Result	Recognition Result	Statistics
a^b	a^b	a^b	$N = 2$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.45$, $C_r = 0.99$
d_c	b_a	p_a	$N = 2$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.31$, $C_r = 0.99$
$b_a\, p^n$	$b_a p^a$	$b_a p^a$	$N = 4$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.45$, $C_r = 0.99$
t_p^m	b_p^a	b_p^a	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.99$
$*v_u^A p_a$	$a_a^b p_a$	$a_a^b p_a$	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.87$
$a_0 h_u^n$	$a_b b_a^a$	$a_b b_a^a$	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.98$
$y_l^n z q_p$	$p_b^a a p_p$	$p_b^a a p_p$	$N = 6$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.41$, $C_r = 0.9$
$b^c pe$	$b^a pa$	$b^a p_a$	$N = 4$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 25\%$ $C_s = 0.39$, $C_r = 0.66$
$\sum_0^n a$	$\sum_b^a a$	$\sum_b^a a$	$N = 4$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.41$, $C_r = 0.94$
$\prod_P^q Y_n$	$\sum_P^P b_a$	$\sum_P^P a_a$	$N = 5$ $e_s = 20\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.39$, $C_r = 0.97$

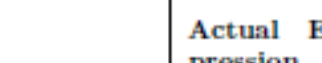
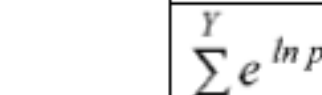
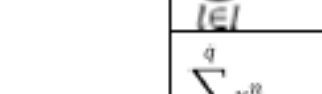
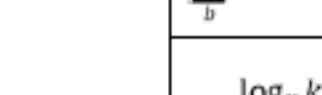
Actual Expression	Expected Result	Recognition Result	Statistics
$cos^2\theta$	$aaa^b b$	$\sum aa^b b$	$N = 5$ $e_s = 20\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.4$, $C_r = 0.96$
$l_o^t so_v c_n^c t^c x_t$	$b_a^b aa_a a_a^a b^a a_b$	$b_{a\sum}^b a_a a_a^a b^a a_b$	$N = 13$ $e_s = 7\%$, $e_p = 15\%$, $e_r = 7\%$ $C_s = 0.46$, $C_r = 0.98$
$*\sum_v^g \prod_0^q t_u r^h$	$\sum_a^P \sum_b^P b_a a^b$	$\sum_a^P \sum_b^F b_a a^p$	$N = 10$ $e_s = 10\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.4$, $C_r = 0.95$
$log_2 x^r$	$bap_b a^a$	bap_{ba^a}	$N = 6$ $e_s = 0\%$, $e_p = 16\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.76$
$\bigcup_H h_m^n \bigcap_t^T x_b$	$\sum_b b_a^a \sum_b^b a_b$	p_{b^{a}_{a} \sum^{a} a_{b}} _{a b}	$N = 10$ $e_s = 30\%$, $e_p = 10\%$, $e_r = 20\%$ $C_s = 0.39$, $C_r = 0.9$
$2\sum_0^Z x\ 42 \prod_v y_n$	$b\sum_b^b abb \sum_a p_a$	b b_{a b b} ^{a}_{b} \sum_{a} p_{a}	$N = 11$ $e_s = 18\%$, $e_p = 9\%$, $e_r = 9\%$ $C_s = 0.38$, $C_r = 0.82$
$\Gamma_x^a \Delta$	$b_a^a b$	$b_a^a a$	$N = 4$ $e_s = 25\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.37$, $C_r = 0.99$
$\bigoplus_x^k \delta_x$	$\sum_a^b b_a$	$\sum_a^P b_a$	$N = 5$ $e_s = 20\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.36$, $C_r = 0.96$
$\psi \bigotimes_w \prod k^p$	$p\sum_a \sum b^p$	$pa_{a\sum\ p^p}$	$N = 6$ $e_s = 33\%$, $e_p = 16\%$, $e_r = 0\%$ $C_s = 0.32$, $C_r = 0.94$
$not \bigwedge_x^U \bigvee_q pq^2$	$aab \sum_a^b \sum_p pp^b$	a a b \sum^{a}_{a} p_{p p^{b}}_{p}	$N = 11$ $e_s = 18\%$, $e_p = 0\%$, $e_r = 9\%$ $C_s = 0.37$, $C_r = 0.8$

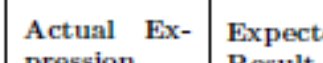
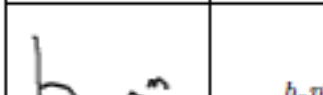
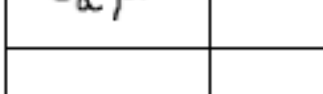
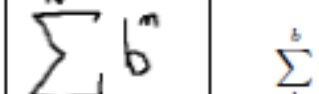
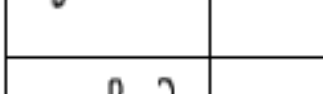
Actual Expression	Expected Result	Recognition Result	Statistics
5^{23}	b^{bb}	b^{bb}	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.46$, $C_r = 0.99$
$ta^{paq}hg$	$ba^{pap}bp$	$ba^{Pap}bp$	$N = 7$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 14\%$ $C_s = 0.4$, $C_r = 0.66$
g_{lu}^{pdw}	p_{ba}^{pba}	p_{ba}^{Ppa}	$N = 6$ $e_s = 16\%$, $e_p = 0\%$, $e_r = 16\%$ $C_s = 0.38$, $C_r = 0.64$
e^{2ln2}	a^{bbab}	a^{bbab}	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.45$, $C_r = 0.59$
$z^{\sum Z}$	$a^{\sum b}$	$a^{a}b$	$N = 3$ $e_s = 33\%$, $e_p = 33\%$, $e_r = 0\%$ $C_s = 0.3$, $C_r = 0.99$
$\lambda n\ 2d_{Hkl}^{3}sn\theta$	$babb_{bbb}^{b}aab$	$pabb_{abb}^{b}aab$	$N = 11$ $e_s = 18\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.38$, $C_r = 0.87$
$a_{long}^{very}ME$	$a_{baap}^{aaap}bb$	$a_{baap}^{aaap}aa$	$N = 11$ $e_s = 18\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.39$, $C_r = 0.9$
$*\hbar\,\partial_{tu}\,\Phi_{x\,t}$	$bb_{ba}b_{ab}$	$pb_{ba}a_{ab}$	$N = 7$ $e_s = 28\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.39$, $C_r = 0.52$
$log_{2x}^{10}x$	$bap_{ba}^{bb}a$	$bapb^{b}a^{ba}$	$N = 8$ $e_s = 0\%$, $e_p = 37\%$, $e_r = 37\%$ $C_s = 0.43$, $C_r = 0.69$
x_{max}	a_{aaa}	a_{aaa}	$N = 4$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.95$

Actual Expression	Expected Result	Recognition Result	Statistics
$\sum_{ln2}^{lnX} e^{snp}$	$\sum_{bab}^{bab} a^{aap}$	$\sum_{bab}^{baa} a^{a}ap$	$N = 11$ $e_s = 9\%$, $e_p = 9\%$, $e_r = 0\%$ $C_s = 0.41$, $C_r = 0.76$
$\sum_{ghm} p$	$\sum_{pba} p$	a_{p}_{p^{p a}}	$N = 5$ $e_s = 40\%$, $e_p = 0\%$, $e_r = 40\%$ $C_s = 0.3$, $C_r = 0.61$
$*e^{\prod lnx}$	$a^{\sum baa}$	$a^{\sum baa}$	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.38$, $C_r = 0.81$
$\sum_{2xy}^{aN} log_{aa}F$	$\sum_{bap}^{ab} bap_{aa}b$	$\sum_{bap}^{aa} bap_{aa}a$	$N = 12$ $e_s = 16\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.41$, $C_r = 0.79$
$*\bigcap_{a}^{Bn}\bigcup_{pP} Y^{m}$	$\sum_{a}^{ba}\sum_{pb} b^{a}$	\sum^{a a}_{a} p_{a^{a}} _{p^{a}}	$N = 9$ $e_s = 44\%$, $e_p = 0\%$, $e_r = 22\%$ $C_s = 0.36$, $C_r = 0.71$
$x^{\gamma u}\prod_{qF}^{Mv} E_{avg}$	$a^{ba}\sum_{pb}^{ba} b_{aap}$	$a^{ba}\sum_{p^{a}}^{aa} a_{a}ap$	$N = 12$ $e_s = 25\%$, $e_p = 8\%$, $e_r = 8\%$ $C_s = 0.41$, $C_r = 0.75$
$K_{\Pi x}^{\prod y}$	$b_{\sum a}^{\sum P}$	$b_{\sum a}^{\sum P}$	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.36$, $C_r = 0.98$
$\sum_{\bigcup A}^{\bigcap d} mv^{2}$	$\sum_{\sum b}^{\sum b} aa^{b}$	$\sum_{Pa}^{Pp} aa^{b}$	$N = 8$ $e_s = 50\%$, $e_p = 0\%$, $e_r = 25\%$ $C_s = 0.34$, $C_r = 0.69$
$\varepsilon_{ap}^{\Gamma\pi}$	a_{ap}^{ba}	a_{ap}^{ba}	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.4$, $C_r = 0.75$
$\varphi\bigvee_{Sv}^{4l}\bigwedge_{0} Tp$	$p\sum_{ba}^{bb}\sum_{b} bp$	$\sum\sum_{a_{a}}\sum_{b}^{bb} ap$	$N = 10$ $e_s = 30\%$, $e_p = 0\%$, $e_r = 20\%$ $C_s = 0.35$, $C_r = 0.81$

Actual Expression	Expected Result	Recognition Result	Statistics
$a^{u^{a}}but_{a^{n}}$	$a^{a^{a}}bab_{a^{a}}$	$a^{a^{a}}bab_{a^{a}}$	$N = 8$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.47$, $C_r = 0.71$
$v^{s^{A}_{a}}r_{p^{n}}$	$a^{a^{b}_{a}}a_{p^{a}}$	$a^{a^{b}_{a}}a_{p^{a}}$	$N = 7$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.43$, $C_r = 0.99$
$ra^{P_{t}\circ}r$	$aa^{P_{b}a}a$	$\sum a^{p_{ba}}a$	$N = 6$ $e_s = 16\%$, $e_p = 16\%$, $e_r = 16\%$ $C_s = 0.38$, $C_r = 0.84$
$*\sum_{a^{b}}^{k_{\circ}} K$	$\sum_{a^{b}}^{b_{a}} b$	$\sum_{a^{b}}^{p_{a}} a$	$N = 6$ $e_s = 33\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.38$, $C_r = 0.98$
$F_{x^{2}}$	$b_{a^{b}}$	$b_{a}b$	$N = 3$ $e_s = 0\%$, $e_p = 33\%$, $e_r = 33\%$ $C_s = 0.38$, $C_r = 0.99$
$S^{k_{y}}$	$b^{b_{p}}$	$\sum^{Fp}$	$N = 3$ $e_s = 66\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.24$, $C_r = 0.99$
$e^{n_{2}}_{u^{7}}$	$a^{a_{b}}_{a^{b}}$	$a^{a_{b}}_{a^{b}}$	$N = 5$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.45$, $C_r = 0.99$
$n^{c^{s^{i_{s_{d}}}}}$	$a^{a^{a^{b_{a_{b}}}}}$	$a^{a^{a^{b_{a_{p}}}}}$	$N = 6$ $e_s = 16\%$, $e_p = 16\%$, $e_r = 16\%$ $C_s = 0.38$, $C_r = 0.95$
$*3x_{s_{0}}\ 2y_{t_{n}}$	$ba_{a_{b}}bp_{b_{a}}$	$ba_{a_{b}}bpb_{a}$	$N = 8$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 12\%$ $C_s = 0.45$, $C_r = 0.73$
$\sum_{\sum_{a}^{b} g}^{\prod_{u}^{v} l} A_{r}$	$\sum_{\sum_{a}^{b} p}^{\sum_{a}^{a} b} b_{a}$	$\sum_{b_{ap}^{b}}^{b_{ab}^{a}} a_{a}$	$N = 11$ $e_s = 27\%$, $e_p = 18\%$, $e_r = 0\%$ $C_s = 0.34$, $C_r = 0.99$

Actual Expression	Expected Result	Recognition Result	Statistics
$d^{a_{23}^{b}}vb_{n}$	$b^{a_{bb}^{b}}ab_{a}$	$b^{a_{bb}^{b}}ab_{a}$	$N = 8$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.46$, $C_r = 0.85$
$e^{\prod_{0}^{n} 2nx}$	$a^{\sum_{b}^{a} baa}$	$a^{b_{bbaa}^{a}}$	$N = 7$ $e_s = 14\%$, $e_p = 14\%$, $e_r = 0\%$ $C_s = 0.4$, $C_r = 0.82$
$D_{c_{ep}}T^{h^{\frac{a_{h}^{u}}{T}^{g}}}$	$b_{a_{a_{b}}}b^{b_{b}^{a^{a^{p}}}}$	$b_{a_{aa}}a^{p^{a_{p}^{b}}}ap$	$N = 11$ $e_s = 54\%$, $e_p = 27\%$, $e_r = 27\%$ $C_s = 0.33$, $C_r = 0.76$
$a^{\Gamma_{\Psi}}$	$a^{b_{a}}$	$a^{b_{a}}$	$N = 3$ $e_s = 0\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.4$, $C_r = 0.99$
$\sum_{\Sigma_{1}}^{\Sigma_{2}} \Pi_{x^{2}}$	$\sum_{b_{b}}^{b_{b}} b_{a^{b}}$	$\sum_{a_{b}}^{a_{b}} a_{a}b$	$N = 8$ $e_s = 37\%$, $e_p = 12\%$, $e_r = 12\%$ $C_s = 0.35$, $C_r = 0.97$
$\bigotimes_{\varphi^{a}}^{\psi^{b}} z_{x}$	$\sum_{p^{a}}^{p^{b}} a_{a}$	$\sum_{a^{a}}^{p^{b}} a_{a}$	$N = 7$ $e_s = 14\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.39$, $C_r = 0.98$
$m_{g^{h}}$	$a_{p^{b}}$	$a_{p^{p}}$	$N = 3$ $e_s = 33\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.33$, $C_r = 0.95$
$b_{c^{m^{0}}}$	$b_{a^{a^{b}}}$	$b_{a}b$	$N = 4$ $e_s = 0\%$, $e_p = 25\%$, $e_r = 25\%$ $C_s = 0.42$, $C_r = 0.99$
$\bigcup_{a^{Y}}^{m} \bigcap_{r_{l}}^{N} \Delta$	$\sum_{a^{b}}^{a} \sum_{a_{b}}^{b} b$	$\sum_{a^{a}}^{a} \sum_{a_{b}}^{a} a$	$N = 9$ $e_s = 33\%$, $e_p = 0\%$, $e_r = 0\%$ $C_s = 0.38$, $C_r = 0.95$
$\varphi_{n} \bigvee_{\bar{p}x}^{9y} X\ Y$	$p_{a} \sum_{p_{a}}^{p_{p}} bb$	$\sum a \sum_{\sum a}^{p_{p}} aa$	$N = 9$ $e_s = 44\%$, $e_p = 11\%$, $e_r = 22\%$ $C_s = 0.34$, $C_r = 0.92$

Actual Expression	Expected Result	Recognition Result	Statistics
	$bbbbabbp$	$\sum aa \sum ab \sum p$	$N = 8$ $e_s = 62\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.31,\ C_r = 0.84$
	$\sum_{a}^{b} a^{bap}$	$\sum_{a}^{a} abaa$	$N = 7$ $e_s = 28\%,\ e_p = 0\%,\ e_r = 14\%$ $C_s = 0.37,\ C_r = 0.58$
	$\sum_{ba}^{p^b} ap$	$\sum_{ba}^{a^b} p_{\Sigma}$	$N = 7$ $e_s = 42\%,\ e_p = 0\%,\ e_r = 14\%$ $C_s = 0.37,\ C_r = 0.8$
	$p^{\mathbb{P}b} b_{a^a}$	$a^{PPP} a^a$	$N = 6$ $e_s = 50\%,\ e_p = 33\%,\ e_r = 33\%$ $C_s = 0.31,\ C_r = 0.99$
	$b_b^{\sum ba}$	a_p^{paa}	$N = 5$ $e_s = 80\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.26,\ C_r = 0.66$
	a^{bb}	a^{bp}	$N = 3$ $e_s = 33\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.33,\ C_r = 0.99$
	$\sum_{bab} b_b$	$b_{bab}^{a_b}$	$N = 6$ $e_s = 33\%,\ e_p = 0\%,\ e_r = 16\%$ $C_s = 0.34,\ C_r = 0.79$
	$\sum_{b}^{P} a^a$	$\sum_{P}^{P} a^a$	$N = 5$ $e_s = 20\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.39,\ C_r = 0.94$
	a^{bapab^a}	a^{bapab^a}	$N = 7$ $e_s = 0\%,\ e_p = 14\%,\ e_r = 14\%$ $C_s = 0.41,\ C_r = 0.88$
	$aaaba$	$papba$	$N = 5$ $e_s = 40\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.35,\ C_r = 0.74$

Actual Expression	Expected Result	Recognition Result	Statistics
	$aaaaba$	$a \sum \sum bb_a$	$N = 6$ $e_s = 50\%,\ e_p = 0\%,\ e_r = 16\%$ $C_s = 0.31,\ C_r = 0.82$
	$b_a p^a$	$b_a p^a$	$N = 4$ $e_s = 0\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.44,\ C_r = 0.83$
	$\sum_{b}^{b} b^a$	$\sum_{p}^{p} b^a$	$N = 5$ $e_s = 40\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.34,\ C_r = 0.9$
	a^{abab}	$a^{ab_p b}$	$N = 5$ $e_s = 20\%,\ e_p = 20\%,\ e_r = 20\%$ $C_s = 0.4,\ C_r = 0.72$
	a^{b^a}	a^{b^a}	$N = 7$ $e_s = 28\%,\ e_p = 0\%,\ e_r = 0\%$ $C_s = 0.39,\ C_r = 0.96$